

Communications presented at SPT2007¹

Mina Abd-el-Malek (American University in Cairo, Egypt)

Internal flow problem through a conducting thin duct via the Lie group method

Summary: The problem of non-linear heat transfer to internal flow subject to an axial variation of the external heat transfer coefficients is assumed. The energy equation in a rectangular/cylindrical coordinate system can be used to find the temperature distribution for flow flowing in a duct with velocity, surrounded by outer surface. We apply Lie-group method for determining symmetry reductions of partial differential equations. Lie-group method starts out with a general infinitesimal group of transformations under which a given partial differential equation is invariant, then, the determining equations are derived. The obtained equations are a set of linear differential equations, the solution of which gives the transformation function or the infinitesimals of the dependent and independent variables. After the group has been determined, a solution to the given partial differential equation may be found from the invariant surface condition such that its solution leads to similarity variables that reduce the number of independent variables in the system. Effect of heat transfer coefficient, fluid velocity and heat generation source on the temperature has been studied and the results are plotted.

Dmitry Alekseevsky (University of Edinburgh, UK)

Homogeneous bi-Lagrangian structures

Summary: A bi-Lagrangian structure on a symplectic manifold (M, ω) is a decomposition $TM = L_+ + L_-$ of the tangent bundle into a direct sum of two integrable Lagrangian distributions L_{\pm} . We give a description of invariant bi-Lagrangian structures on a (real or complex) symplectic homogeneous manifold $M = G/K$ of a semisimple Lie group G in terms of crossed Dynkin and Satake diagrams. Some applications will be discussed. This is a joint work with C. Medori (Parma).

Nora Alexeeva (University of Cape Town, South Africa)

No communication presented

Ilhame Amiraliyeva (Yuzuncu Yil University, Van, Turkey)

No communication presented

Konstantionos Andriopoulos (Patras University, Greece)

The Complete Symmetry Group story

¹NCE = no communication expected

Summary: The notion of complete symmetry groups is revisited and extended in the sense that it refers to those groups of symmetries that completely specify classes of equations/systems. The terminology, Lie remarkable equations, is indeed neat with a limited occurrence in differential equations since the only admissible symmetries are Lie point. Lie remarkability is straightforwardly embodied into complete symmetry groups. The need for nonlocal symmetries for the complete specification of differential equations is highlighted with the use of an example taken from the recent literature on differential sequences.

Mayeul Arminjon (CNRS et Université de Grenoble, France)

Quantum wave equations in curved space-time from wave mechanics

Summary: The standard wave equations in a curved space-time are inspired by the equivalence principle. One looks for a covariant equation that coincides with the flat-space-time equation if, at the event considered, the metric tensor has the flat form and the relevant connection cancels. For the Dirac equation, this leads to the Dirac-Fock-Weyl (DFW) equation. However, the latter is based on the “spin connection”, which does not cancel in a local freely falling frame (in which the metric connection cancels). Thus, the DFW equation does not obey the genuine equivalence principle, which inspires its derivation. We propose a derivation of the Klein-Gordon (KG) and Dirac equation in a curved space-time, directly from the principles of wave mechanics. This leads to two new gravitational Dirac equations: one does obey the equivalence principle, the other one has a preferred reference frame. Each of them is compatible with the corresponding KG equation.

Esmaeel Asadi (Vrije Universiteit Amsterdam, Netherlands)

Integrable systems of geometric curves

Summary: The evolution of invariants of a curve embedded in a homogeneous space and geometric operators attached to that will be discussed

Angel Ballesteros (Universidad de Burgos, Spain)

Superintegrable spaces of non-constant curvature

Summary: N -dimensional spaces endowed with coalgebra symmetry are introduced. For all these spaces the geodesic flows are shown to be quasi-maximally superintegrable, and the full set of constants of the motion is explicitly obtained from the underlying coalgebra. As remarkable non-constant curvature examples of this construction, the ND generalization of the four 2D Darboux spaces is presented, together with a new ND space coming from a quantum deformation of $sl(2)$.

Igor Barashenkov (University of Cape Town, South Africa)

Interaction of Domain Walls in an Anisotropic XY Model

Summary: The dynamics of magnetization in the anisotropic XY model is described by the “repulsive” (or “defocusing”) parametrically driven nonlinear

Schroedinger equation, $i\Psi_t + (1/2)\Psi_{xx} + (1 - |\Psi|^2)\Psi = h\Psi^*$. The Bloch and Ising walls are two coexisting stable soliton solutions of this equation. We study the interaction of the two walls by following the dissociation of an unstable Bloch-Ising bound state. Mathematically, the analysis focusses on the splitting of a four-fold zero eigenvalue associated with a pair of infinitely separated Bloch and Ising walls. The lifting of the degeneracy of a repeated eigenvalue of the scalar Schroedinger operator with the potential comprising two identical potential wells with large separation, is discussed in standard textbooks on quantum mechanics. This analysis is not helpful in our case, unfortunately, for three reasons: (i) our linearised operator operates on vector, not scalar, functions; (ii) the potential wells formed by the Bloch and Ising walls are not identical; (iii) the textbook analysis postulates a particular form of the wavefunction on symmetry grounds, rather than deriving it within some perturbation formalism — as a result, the generalization to the vector nonsymmetric case is not straightforward. Our treatment is based on expanding the eigenfunction in the asymptotic series near the cores of the two walls and matching the resulting expansions in the overlap region. We show that a Bloch and an Ising wall interact as two classical particles, one with positive and the other one with negative mass. This is a joint work with S. Woodford (Jülich).

Vivina Barutello (Università di Milano-Bicocca, Italy)

On the singularities of generalized solutions to n-body type problems

Summary: The validity of Sundman-type asymptotic estimates for collision solutions is established for a wide class of dynamical systems with singular forces, including the classical n-body problems with Newtonian, quasi-homogeneous and logarithmic potentials. The solutions are meant in the generalized sense of Morse (locally –in space and time– minimal trajectories with respect to compactly supported variations) and their uniform limits. The analysis includes the extension of the Von Zeipel’s Theorem and the proof of isolatedness of collisions. Furthermore, such asymptotic analysis is applied to prove the absence of collisions for locally minimal trajectories.

Peter Basarab-Horvath (Linköping University, Sweden)

Symmetry Classification of PDEs

Summary: General classes of PDEs may be classified using the usual Lie symmetry approach combined with the equivalence groups of the type of equations at hand. We give a short survey of some of the results obtained and some of the techniques involved.

Sergio Benenti (Università di Torino, Italy)

An algorithm for the computation of the curvature tensors without the Christoffel symbols

Summary: The Riemann tensor of a given metric of any dimension and signature can be computed ‘by hand calculation’, avoiding the explicit calculation of the

Christoffel symbols. The algorithm is based on the Lagrange geodesic equations and it can be used for compiling a fast software.

Luca Biasco (Università di Roma Tre, Italy)
Birkhoff–Lewis type results for the nonlinear wave equation

Summary: We show some recent results on the existence of infinitely many periodic solutions accumulating to zero for the one–dimensional nonlinear wave equation (vibrating string equation).

Sergey Bolotin (University of Wisconsin, Madison, USA)
Second species solutions of the elliptic 3 body problem

Summary: Not provided

Henk W Broer (University of Groningen, Netherlands)
Quasi-periodicity in dynamical systems

Summary: The interest is formed by multi- or quasiperiodic motions in dynamical systems as these occur in a ‘visible and robust’ way. Such motions involve more than one frequency and often form an intermediate stage between periodic and chaotic dynamics. Quasi-periodicity first was ‘discovered’ in Hamiltonian systems, used for the modelling within frictionless mechanics, and – among many other things – has been related to the eternal stability of the Solar system. The technique was developed by Kolmogorov-Arnold-Moser (KAM) and was later extended to other classes of systems, like reversible or dissipative systems. In the latter case there is a persistent occurrence of families of quasi-periodic attractors, that play a role in the Hopf-Landau-Lifschitz-Ruelle-Takens theory around the onset of turbulence. In the talk we sketch this KAM theory also pointing at some recent developments.

Francesco Calogero (Università di Roma “La Sapienza”, Italy)
Isochronous systems are not rare

Summary: A dynamical system is called isochronous if it features an open (hence fully dimensional) region in its phase space in which all its solutions are completely periodic (i. e., periodic in all degrees of freedom) with the same fixed period (independent of the initial data, provided they are inside the isochronicity region). A trick is presented associating to an (autonomous) dynamical system an (also autonomous) modified system depending on a real parameter so that when this parameter vanishes the original system is reproduced while when this parameter does not vanish the modified system is isochronous. This technique is applicable to large classes of dynamical systems, justifying the title of this talk. Analogous techniques, even more widely applicable - for instance, to any translation-invariant (classical) many-body problem - transform a Hamiltonian system into an isochronous Hamiltonian system. Examples and (Diophantine) findings arrived at in this manner will be presented. These techniques can also be applied to PDEs - but there will be no time to explore this extension.

Diego Catalano Ferraioli (Università di Milano, Italy)

Nonlocal aspects of λ -symmetries and ODEs reduction

Summary: A reduction method of ODEs not possessing Lie point symmetries makes use of the so called λ -symmetries (C. Muriel and J. L. Romero). The notion of covering for an ODE \mathcal{Y} is used here to recover λ -symmetries of \mathcal{Y} as nonlocal symmetries. In this framework, by embedding \mathcal{Y} into a suitable system \mathcal{Y}' determined by the function λ , any λ -symmetry of \mathcal{Y} can be recovered by a local symmetry of \mathcal{Y}' . As a consequence, the reduction method of Muriel and Romero follows from the standard method of reduction by differential invariants applied to \mathcal{Y}' .

Martin Celli (Scuola Normale Superiore, Pisa, Italy)

The central configurations of four masses $x, -x, y, -y$.

Summary: The configuration of a homothetic motion in the N-body problem is called a central configuration. We prove that there are exactly three planar non-collinear central configurations for masses $x, -x, y, -y$ with $x < y$ (a parallelogram and two trapezoids) and two planar non-collinear central configurations for masses $x, -x, x, -x$ (two diamonds). Except these cases, the only known case where the four-body central configurations with non-vanishing masses can be listed is the case with equal masses (A. Albouy, 1996), which requires the use of a symbolic computation program.

Mark Chanachowicz (University of Waterloo, Canada)

Invariant classification of rotationally symmetric conformal Killing tensors in E^3

Summary: The valence two conformal Killing tensors in E^3 admitting rotational symmetry are classified under the action of the conformal group. The relationship with the rotationally symmetric orthogonally R -separable webs of Laplace's equation is elucidated. Joint work with C. Chanu and R. McLenaghan.

Claudia Chanu (Università di Torino, Italy)

R-separation of the conformal invariant Laplace equation

Summary: The conformal invariant Laplace equation is a generalization of the well-known Laplace equation on a flat n-dimensional Riemannian manifold, which is invariant with respect conformal transformation of the metric. The conditions for fixed energy R-separation of this equation are analyzed and in particular the three dimensional case is described in details. This is a joint work with R.G. McLenaghan and M. Chanachowicz.

Anna Maria Cherubini (Università del Salento, Lecce, Italy)

No communication presented

Luigi Chierchia (Università di Roma Tre, Italy)

Quasi-periodic attractors in celestial mechanics

Summary: We discuss under which conditions, in a weakly dissipative regime (as in the case of celestial mechanics) KAM tori bifurcate in a regular way into quasi-periodic attractors. We focus on a concrete model arising from celestial mechanics, i.e. the dissipative spin-orbit model, with the (“averaged MacDon-ald”) dissipation function recently considered by Correia and Laskar.

Giampaolo Cicogna (Università di Pisa, Italy)

No communication presented

Giuseppe M Coclite (Università di Bari, Italy)

Discontinuous solutions for the Degasperis-Procesi equation

Summary: We consider the nonlinear third order dispersive Degasperis-Procesi equation, that models shallow water dynamics with asymptotic accuracy which is one order more accurate than the KdV one. We are interested in the Cauchy problem for this equation, so we augment it with an initial condition $u_0 \in L^1(\mathbb{R}) \cap BV(\mathbb{R})$. Formally, the problem is equivalent to an hyperbolic-elliptic system or to a conservation law with a nonlocal flux or source function. We prove the existence and uniqueness of the entropy weak solution, that is a distributional solution satisfying some additional entropy conditions. In addition the unique entropy weak solution satisfies the Oleinik type estimate; an implication is that a shock wave in an entropy weak solution to the Degasperis-Procesi equation is admissible only if it jumps down in value (like the inviscid Burgers equation).

Vittorio Coti-Zelati (Università di Napoli, Italy)

Variational methods and homoclinic solutions to invariant tori

Summary: I will present some results, obtained (in collaboration with M. Macrì) using variational methods, about existence of solutions homoclinic to periodic orbits and two dimensional invariant tori in a centre manifold.

Antonio Degasperis (Università di Roma “La Sapienza”, Italy)

Darboux dressing construction of solutions to integrable PDEs with nonvanishing boundary values

Summary: Not provided

Florin Diacu (University of Victoria, Canada)

Saari’s Homographic Conjecture of the 3-Body Problem

Summary: Saari’s homographic conjecture, which extends a classical statement proposed by Donald Saari in 1970, claims that solutions of the Newtonian n -body problem with constant configurational measure are homographic. In other words, if the mutual distances satisfy a certain relationship, the configuration of the particle system may change size and position but not shape. We prove this conjecture for large sets of initial conditions in three-body problems given

by homogeneous potentials, including the Newtonian one. Some of our results are true for $n \geq 3$.

Oana M Dragulete (Ecole Polytechnique Federale de Lausanne, Switzerland)
No communication presented

Jacques Féjoz (Université P. & M. Curie, Paris, France)
Unchained polygons and the n -body problem

Summary: Look to a relative equilibrium of the n -body problem in a rotating frame which puts into resonance the frequency of a relative equilibrium of the n -body problem and that of an infinitesimal variation normal to the plane of the equilibrium. Continuation with respect to the rotating velocity of the frame yields a remarkable class of periodic solutions. The first example is the P_{12} family, discovered by C. Marchal, which links the relative equilibrium of Lagrange to the Eight. Other examples are given by the continuation of the n -gon.

Mark E Fels (University of Utah, Logan, USA)
Darboux integrable Harmonic maps

Summary: This talk will demonstrate the theory of Darboux Integrability for systems of partial differential equations using sigma models (or harmonic maps). Some classifications will be given if time permits.

Davide L Ferrario (Università di Milano-Bicocca, Italy)
Transitive decomposition of n -body symmetry groups

Summary: We show under which extent equivariant minimizers of the n -body Lagrangean action functional are in fact collisionless periodic orbits, and show how to generate and classify all such symmetry groups. To this aim it is necessary to survey in this setting some asymptotic results on collisions and a new averaging method used for the construction of local variations. We illustrate the method for few bodies and show how to apply it to computer generated numerical simulations.

Giuseppe Gaeta (Università di Milano, Italy)
No communication presented

Valentina Golovko (Lomonosov Moscow State University, Russia)
Nonlocal Poisson-Nijenhuis structures

Summary: Poisson-Nijenhuis structures in the case of nonlocal differential operators in total derivatives are considered and the compatibility conditions for a Hamiltonian operator and a recursion operator with zero Nijenhuis torsion is obtained. All the conditions are expressed in terms of the Jacobi bracket for shadows of symmetries (constructed as well) while Hamiltonian operators and recursion operators are treated as shadows in the corresponding coverings.

M. Alejandra Gonzalez-Enriquez (Università di Camerino, Italy)

Analytic smoothing of geometric maps with applications to KAM theory

Summary: We prove that finitely differentiable symplectic, volume-preserving, or contact diffeomorphisms can be approximated in analytic norms with symplectic, respectively volume-preserving or contact, analytic diffeomorphisms. The quality of the approximation is given in terms of the order of differentiability of the approximated diffeomorphism. As an application to this result, a proof of a finitely differentiable version of a KAM theorem is given. The bootstrap of regularity of invariant tori for exact symplectic maps is also proved.

Todor Gramchev (Università di Cagliari, Italy)

Divergent normal forms in Gevrey spaces

Summary: The talk will outline some techniques for studying the Gevrey type of divergent normal forms for analytic vector fields near singular points and analytic Hamiltonian systems

Giovanni Federico Gronchi (Università di Pisa, Italy)

Predetermination of orbital elements

Summary: Not provided

Faruk Gungor (Istanbul Technical University, Turkey)

Symmetries of generalized Davey-Stewartson equations

Summary: We study symmetries of generalized Davey-Stewartson equations and show that they have loop structures for specific choices of constants appearing in the coefficients. We also look at the possibility of transforming the system to the usual Davey-Stewartson system. Finally, we discuss the blow-up behavior of solutions using pseudo-conformal symmetries as finite-dimensional subgroups of the entire symmetry group.

Heinz Hansmann (University of Utrecht, Netherlands)

On the destruction of resonant Lagrangian tori in Hamiltonian Systems

Summary: Poincaré's fundamental problem of dynamics concerns the behaviour of an integrable Hamiltonian system under a (small) non-integrable perturbation. Under rather weak conditions Kolmogorov-Arnol'd-Moser theory settles this question for the majority of initial values. The perturbed motion is (again) quasi-periodic, the number of frequencies equals the number of degrees of freedom. KAM theory proves such Lagrangean tori to persist provided that the frequencies are bounded away from resonances by means of Diophantine inequalities. How do Lagrangean tori with resonant frequencies behave under perturbation? We concentrate on a single resonance, whence many n -parameter families of n -tori are expected to be generated by the perturbation ; here $n+1$ is the number of degrees of freedom. For non-degenerate systems we explain the pattern how these families of lower-dimensional tori come into existence, and then discuss what happens in the presence of degeneracies.

Joshua Horwood (Corpus Christi College, Cambridge, UK)

On the computation of first integrals of motion cubic in the momenta

Summary: In this talk, we shall present a direct and systematic method for computing first integrals of motion which are cubic in the momenta for natural Hamiltonian systems defined in a flat space of arbitrary dimension. We will then apply our method to explore the integrability of the Calogero-Moser system, its generalizations and the class of rotationally symmetric potentials in three-dimensional Euclidean space. The talk will culminate in the presentation of a new maximally super-integrable case.

Masatomo Iwasa (Nagoya University, Japan)

A method of asymptotic solutions invariant under the renormalization group

Summary: A renormalization group method with Lie symmetry is presented for the singular perturbation problems. Asymptotic solutions are obtained as group-invariant solutions under approximate Lie group admitted by perturbed ordinary differential equations. If time is left, I will also talk about the application of the method to difference equations and partial differential equations.

Josif Janyška (University of Brno, Czech Republic)

Utiyamas reduction method and infinitesimal symmetries of invariant Lagrangians

Summary: We generalize the Utiyama reduction method for invariant Lagrangians of gravitational and gauge fields and describe infinitesimal symmetries of such Lagrangians.

Boris Konopelchenko (Università del Salento, Lecce, Italy)

Coisotropic and quantum deformations of associative algebras and integrable systems

Summary: Not provided

Ivan Kosenko (Moscow State University of Service, Cherkizovo–Pushkino, Russia)

Stability of the Tethered Satellite System Relative Equilibria. Unrestricted Problem

Summary: The spatial problem of stability for the tethered satellite system (TSS) relative equilibria is resolved completely. The problem is considered in its unrestricted form: material endpoints, the orbital station and the sub-satellite, composing the TSS, supposed moving independently in field of gravity of the fixed attracting center each thus performing the piecewise Keplerian motion. This motion is interrupted from time to time by the unilateral constraint, ideal massless inextensible tether of constant length. Initially an impacts are supposed purely elastic ones. Then the mechanical system becomes a natural one with the discontinuities on velocities. As usual in celestial mechanics the dynamical system has two known integrals: one of energy and of angular momentum.

The simple Routh procedure of reduction similar to node elimination in the three-body problem is performed. Then the only potentially stable radially oriented relative equilibria are computed, and the Ivanov theorem on stability of an equilibrium for the Lagrangian mechanical system with the unilateral constraints is applied. All possible the endpoints mass ratios and the tether lengths are analyzed.

Iosif Krasil'shchik (Independent University of Moscow, Russia)

Nonlocal geometry od PDEs and integrability

Summary: We discuss relations of nonlocal constructions in geometry of PDEs to integrability properties.

Boris Kruglikov (University of Tromso, Norway)

Applications of multi-brackets to formal integrability

Summary: Multi-brackets is an algebraic operation on vector-valued non-linear differential operators, which is skew-symmetric and satisfies certain Plucker identities. I will explain a criterion of formal integrability via multi-brackets (joint with V.Lychagin). As an application I will invariantly characterize Liouville metrics on surfaces and more generally metrics with integrals polynomial in momenta. This leads to a generalization of Darboux-Koenig theorem.

Demeter Krupka (Palacki University, Olomouc, Czech Republic)

Natural variational principles

Summary: A natural variational principle is defined by means of a Lagrangian which is a differential invariant on a natural fibre bundle. We introduce basic notions of the theory and study relations between the corresponding Euler-Lagrange equations and conservation laws.

Olga Krupkova (Palacki University, Olomouc, Czech Republic)

Variational exterior differential systems

Summary: We introduce exterior differential systems associated with Euler-Lagrange equations on fibred manifolds and investigate their symmetries and conservation laws. We also consider the inverse problem of the calculus of variations for exterior differential systems.

Sara Lombardo (Vrije Universiteit Amsterdam, Netherlands)

Algebraic reductions of integrable equations and Automorphic Lie Algebras

Summary: In this talk I will focus on the problem of (algebraic) reductions, one of the most important and challenging problems in the theory of integrable systems. In particular, I will show how this is related to a class of infinite dimensional Lie algebras, which has applications in the theory of integrable equations and beyond. The construction of these algebras is very similar to the one for automorphic functions and this motivates the name automorphic Lie algebras.

Franco Magri (Università di Milano-Bicocca, Italy)

Cyclic systems of Levi Civita

Summary: I will introduce the concept mentioned in the title, and show how it helps in understanding the evolution of integrable systems theory from Lagrange to Whitham.

Gianni Manno (Università del Salento, Lecce, Italy)

Projectively equivalent metrics: a solution of a S. Lie problem

Summary: We give a disjoint list of all 2-dimensional metrics admitting (at least) two-parameter group of geodesic-preserving (local) transformations. This solves the problem posed by S. Lie in 1882.

Krzysztof Marciniak (Linköping University, Sweden)

Separation curves, soliton hierarchies, bi-cofactor systems and geodesic equivalence

Summary: In this talk I shall present various connections between finite- and infinite-dimensional integrable and separable systems originating from separation curves (separation relations). I shall first discuss how the so-called bi-cofactor systems originate from some natural separation curves and relate it to the classical geodesic equivalence (correspondence) problem in the sense of Levi-Civita. In the next part of my talk I shall show that separation curves are in a sense a source of a vast number of soliton hierarchies and that these soliton hierarchies inherit Hamiltonian formulation from the underlying spectral curves.

Luigi Martina (Università del Salento, Lecce, Italy)

Symmetry group and Symplectic Structure for exotic particles in the plane

Summary: Not provided

Vladimir Matveev (University of Jena, Germany)

Geodesically equivalent metrics “in the large”: Beltrami and Schouten problems

Summary: Two metrics are geodesically equivalent if they have the same (unparametrized) geodesics. During my talk I describe geodesically equivalent metrics on closed manifolds (which is an answer to Beltrami’s question) and explain the proof of Lichnerowicz-Obata conjecture (which is an answer on the infinitesimal version of the Beltrami question known as Schouten problem). The methods are the mixture of technique from integrable systems and geometric theory of PDEs.

Ray McLenaghan (University of Waterloo, Canada)

Orthogonally separable webs for the Hamilton-Jacobi equation in three dimensional Minkowski space

Summary: The orthogonally separable webs and associated adapted coordinate systems for the Hamilton-Jacobi equation in three dimensional Minkowski equation are obtained via Eisenhart's integrability conditions for the valence two Killing tensor equations. The procedure is illustrated by the determination for some of the cases of the transformations from canonical pseudo-Cartesian coordinates to canonical separable coordinates and the associated characteristic Killing tensors. (Joint work with J. Horwood)

Andrey Mironov (Sobolev Institute of Mathematics, Novosibirsk, Russia)
On the Tzitzeica equation symmetries' relation to the Novikov-Veselov hierarchy

Summary: We show that the two-dimensional Schroedinger operator naturally connects to the Tzitzeica equation and that its iso-spectral deformations given by the Novikov-Veselov hierarchy correspond to the symmetries of this equation.

Richard Montgomery (University of California at Santa Cruz, USA)
Classical Newtonian N-body Scattering

Summary: For the positive energy Newtonian N-body problem perhaps the simplest solutions are the strictly hyperbolic ones. The bodies come in from infinity at positive speeds, interact briefly, then recede outwards again, each body asymptotic to a different ray. Chazy worked out precise asymptotics of such solutions. Given a set of N distinct rays for the distant past, and another set of such rays for the distant future, does there exist a solution connecting the set of past rays to the set of future rays? We will sketch how gluing methods borrowed from nonlinear elliptic PDE theory yield an affirmative answer. We also discuss the failure of naive applications of the variational methods to answer this existence question affirmatively, and some ideas for possible variational successes.

Paola Morando (Politecnico di Torino, Italy)
Deformation of Lie derivative and μ -symmetries

Summary: We introduce, in the spirit of Witten's gauging of exterior differential, a deformed Lie derivative that allows a geometrical interpretation of λ and μ -symmetries, in complete analogy with standard symmetries. The case of variational symmetries (both for ODEs and for PDEs) is also considered in this approach, leading to λ and μ -conservation laws.

Oleg Morozov (Moscow State Technical University of Civil Aviation, Russia)
Maurer-Cartan forms for symmetry pseudo-groups and coverings of differential equations

Summary: The aim of my talk is to reveal a relationship between two important constructions in geometry of differential equations: Lie symmetry pseudo-groups and coverings (or zero-curvature representations, or Lax pairs, or Wahlquist-Estabrook prolongation structures). I demonstrate that the known coverings for Liouville's equation, the Boyer-Finley equation, and the Khokhlov-Zabolotskaya equation can be defined by invariant combinations of the Maurer-Cartan forms

of their symmetry pseudo-groups. Also, I apply Cartan's method of equivalence to find new covering equations for the modified Khokhlov-Zabolotskaya equation and a two-dimensional deformation of the generalized Hunter-Saxton equation.

Olena Mul (University of Aveiro, Portugal)

Perturbation Theory Method for Analysis of Vibrations in Some Transmission Pipeline Systems

Summary: The perturbation theory method was applied for analysis of possible vibrations in some transmission pipelines intended for lifting of minerals from great depth. A mathematical model of the transmission pipeline with a Lanchester damper was formulated, where the system was considered as a complex dynamical one with distributed and discrete parameters. The proposed model consisted of a partial differential equation and nonlinear boundary conditions. The conditions of vibration excitation as well as a formula for amplitudes of possible vibrations were obtained in analytical form by the perturbation theory method. The important conclusion was drawn that some boundary frequency always existed for considered transmission pipelines that on the frequencies, which were greater than the boundary one, excitation of vibrations was impossible. We showed also that it was possible to change purposefully frequencies and amplitudes of vibrations and avoid an excitation of undesirable vibration processes.

Nikolai N Nekhoroshev (Moscow University, Russia & Università di Milano, Italy)

Fuzzy fractional monodromy

Summary: Not provided

Lena Noethen (Rheinisch Westfaelische Technische Hochschule Aachen, Germany)

Nearly invariant sets

Summary: Reaction systems, possibly of high dimension, contain unknown parameters. A direct analysis could be hard. A reduction of the dimension is one possibility to simplify the analysis. Different approaches are known and possible. In this talk I will introduce and discuss an elementary notion of "nearly invariant sets" for vector fields which is motivated by quasi-steady state phenomena in biochemical reaction systems. If we assume that a reaction proceeds in quasi-steady state (perhaps from insight of biochemists), then we are able to find necessary conditions on the parameters. If we consider a chemical reaction, these parameters consist not necessary only of initial concentrations of the reaction, but rate constants can also be involved. On the other hand, if we start with the assumption that a small parameter is given, there exist a procedure to find iteratively a lower dimensional manifold subject to the small parameter, which is "nearly-invariant". Our method makes it possible to find such a manifold. On the other hand the assumed quasi-steady state could be described more precisely. An example will be the Michaelis/Menten enzyme-catalyzed reaction. In

many cases singular perturbation theory (Tikhonov's theorem) is used. But a detailed analysis shows neither all hypotheses are satisfied to apply the theorem in the case of Michaelis and Menten, nor does the conclusion hold. The problem in a biochemical context is, that we first need to find a small parameter for the system, before we can start the analysis. Our approach is quite adequate for biochemical reactions and relatively easy to work with. This is a starting point for further analysis.

Francesco Oliveri (Universita di Messina, I)

Differential equations characterized by Lie point symmetries

Summary: Within the context of the inverse Lie problem the question whether there exist PDEs that are characterized by their Lie point symmetries may be addressed. Various examples of Lie remarkable equations (either strongly or weakly) exist, including some multidimensional Monge-Ampère type equations. Also, it is shown how to construct Lie remarkable equations determined by some relevant algebras of Lie point symmetries.

Peter J Olver (University of Minnesota, Minneapolis, USA)

Differential Invariant Algebras

Summary: I will present some new results on the structure of differential invariant algebras for both Lie groups and Lie pseudo-groups.

Oliver Oxtoby (University of Cape Town, South Africa)

Moving solitons in the discrete nonlinear Schroedinger equation

Summary: We use the method of asymptotics beyond all orders to evaluate the amplitude of radiation from a moving soliton in the discrete nonlinear Schroedinger equation. When the nonlinearity is of the cubic type, this amplitude is found to be nonzero for all velocities and therefore solitons moving without emitting radiation do not exist, at least in the small-amplitude limit. However, in the case of a *saturable* nonlinearity, the radiation disappears when the soliton moves at certain 'sliding velocities'. We show that a soliton moving at a general speed will experience radiative deceleration until it either stops and remains pinned to the lattice, or locks onto one of the sliding velocities.

Francesco Paparella (Università del Salento, Lecce, Italy)

Asymptotic behavior of a rebounding ball

Summary: Current models of a rebounding object assume that the object is a rigid body, that the impacts are instantaneous, and that the ratio of the normal component of the velocity before and after the impact is a well-defined restitution coefficient. These models are subject to "inelastic collapse": according to them, the object performs infinite bounces in a finite time. Beyond the collapse time the models become meaningless. We present a simple model of an elastic bouncing ball that, for large impact velocities, approximates the behavior of a conventional rigid-body model with constant restitution coefficient. With

careful asymptotic expansions, we prove rigorously that the model is well-posed at all times. We show that the rebounding ball spontaneously passes from a regime of bounces separated by macroscopic flights, to an asymptotic regime of microscopic vibrations. The asymptotic rate of damping of these vibration is computed explicitly. Finally we discuss the results of numerical simulations, in comparison with laboratory experiments, and discuss the applicability of the model to the study of granular materials.

Ernesto Perez-Chavela (Universidad Autonoma Metropolitana – Iztapalapa, Mexico)

Periodic orbits for anisotropic perturbations of the Kepler problem

Summary: We study the Kepler problem perturbed by an anisotropic term, that is a potential conformed by a Newtonian term, $1/r$, plus an anisotropic term, $b/(r^2[1 + \varepsilon \cos^2 \theta])^{\beta/2}$. Because of the anisotropic term, although the system is conservative the angular momentum is not a constant of motion. In this work we present an analytic and numerical analysis for the periodic orbits of a particle moving under the influence of the above potential. For the particular case of $\beta = 2$, there is a second constant of motion, so the system is integrable. We present comparative results of the integrable case $\beta = 2$, and the cases $\beta = 1$ and $\beta = 3$.

Diogo Pinheiro (University of Porto, Portugal)

Interaction of two charges in a uniform magnetic field

Summary: We study the interaction of two charges under the action of a uniform magnetic field. We first look at the planar problem, i.e. the two charged particles move in R^2 . In this setting, we formulate the problem as a four degrees of freedom Hamiltonian system. We prove that this system can always be reduced to one with two degrees of freedom. Furthermore, we identify (distinct) sets in the space of parameters where: a) the system is integrable; b) the system is chaotic. Based on the properties of the reduced system we provide a detailed qualitative description of the original four degrees of freedom system. For the analysis of the spacial problem, we use similar techniques to reduce the dynamics and obtain a asymptotic description for the scattering map associated with this problem. This is joint work with R.S. MacKay, University of Warwick.

Juha Pohjanpelto (Oregon State University, Corvallis, USA)

Differential Invariants for Lie Pseudogroups

Summary: I will report on my ongoing joint work with P. Olver on developing systematic and constructive algorithms for analyzing the structure of continuous pseudogroups and identifying various invariants for their action. In our approach we employ moving frames, which for general pseudogroup actions are defined as equivariant mappings from the space of jets of submanifolds into the pseudogroup jet bundle. The existence of a moving frame requires local freeness of the action in a suitable sense and, as in the finite dimensional case, moving

frames can be used to systematically produce complete sets of differential invariants and invariant coframes for the pseudogroup action and to effectively analyze their algebraic structure. In this talk I will focus on the method of moving frames combined with techniques from commutative algebra to discuss Tresse-Kumpera type existence results for differential invariants of a pseudogroup action and to describe methods for analyzing the algebraic structure of such invariants. Our constructions are equally applicable to finite dimensional Lie group actions and provide a slight generalization of the classical moving frame methods in this case.

Alessandro Portaluri (Università di Milano Bicocca, Italy)

No communication presented

Barbara Prinari (Università del Salento, Lecce, Italy)

No communication presented

Michela Procesi (Università di Roma Tre, Italy)

Periodic solutions for nonlinear dispersive PDE's in $d > 1$ spatial dimensions

Summary: I will consider the problem of existence of periodic solutions for nonlinear dispersive PDE's in $d > 1$ spatial dimensions; in particular I will concentrate on the small divisor problem, which I will solve by using a constructive “renormalization group” approach.

Giuseppe Pucacco (Università di Roma “Tor Vergata”, Italy)

Separation of variables on the hyperbolic plane

Summary: Not provided

Fabrizio Pugliese (Università di Salerno, Italy)

On a special class of Monge-Ampere equations

Summary: We present the first results of a work on classification of parabolic Monge-Ampere equations. This work is in collaboration with R. Alonso Blanco, G. Manno and A.M. Vinogradov

Orlando Ragnisco (Università di Roma Tre, Italy)

Integrable models on curved space from q -algebras: equations of motion and their solution

Summary: Not provided

Giovanni Rastelli (Università di Torino, Italy)

Decomposition of scalar potentials of natural Hamiltonians into integrable and “perturbative” terms. A naive approach

Summary: The geometry of Stackel systems is employed to find, for a given natural Hamiltonian, another natural Hamiltonian which is integrable by separation of variables and in some sense close to the first one. The study is intended

as a first step for a perturbative analysis of a given Hamiltonian system. The method is coordinate independent and effective mainly on manifolds of constant curvature. Examples are given for the quadrupole field and Henon-Heiles system.

Stefan Rauch-Wojciechowski (Linköping University, Sweden)

Separation of potential and quasi-potential Newton equations

Summary: Theory of quasipotential Newton equations studies equations $d^2q/dt^2 = M(q)$ admitting an energy-like integral of motion which depends quadratically on velocities. If $d^2q/dt^2 = M(q)$ admits two such integrals then there are n quadratic integrals of motion and these Newton equations are completely integrable and separable in a non-standard sense. They can be characterized through a certain Poisson pencil or through a system of second order linear partial differential equations called Fundamental Equations. This theory appears to be a natural extension of the classical separability theory (for the Helmholtz equation and for the Hamilton-Jacobi equation) but it covers other types of equations such as triangular equations and driven equations. Examples of new non-orthogonal separation variables will be presented.

Omar Rojas (La Trobe University, Melbourne, Australia)

Closed form expression for integrals of sG map

Summary: We present closed-form expressions for approximately N integrals of the $2N$ -dimensional map. The mapping is obtained by travelling wave reductions of the integrable discrete sine-Gordon equation. We provide the integrating factors corresponding to the integrals. Moreover we show how the integrals and the integrating factors relate to the staircase method of Quispel et al.

Alexander Rutherford (Simon Fraser University, Vancouver, Canada)

Pseudorotational Spectra of Molecules and Isoparametric Geometry

Summary: We consider the problem of calculating the pseudorotational energy levels associated with the molecular Jahn-Teller effect. The focus is on the strong Jahn-Teller coupling limit for electronic triplets of octahedral, tetrahedral, or icosahedral molecules. We show that the energy levels are related to the spectra of Laplace-Beltrami operators on vector bundles over the leaves of Cartan's isoparametric foliation of S^4 . These spectra are computed explicitly using the Peter-Weyl theorem for homogenous spaces and harmonic analysis on vector bundles. We also explore the spectral flow transverse to the leaves of the foliation.

Giuseppe Saccomandi (Università del Salento, Lecce, Italy)

A general reduction method for finite amplitude elastic waves

Summary: In this talk I study the interaction of a longitudinal wave with transverse waves in general isotropic and unconstrained hyper-elastic materials, including the possibility of dissipation and dispersion. The goal is to derive the

corresponding general equations of motion, valid for any possible form of the strain energy function and to investigate the possibility of obtaining some general and exact solutions to these equations by reducing them to a set of ordinary differential equations. The solutions derived are in a time-space separable form and may be interpreted as generalized oscillatory shearing motions and generalized sinusoidal standing waves. By means of standard methods of dynamical systems theory, some peculiar properties of waves propagating in compressible materials are uncovered, such as for example, the emergence of destabilizing effects.

Vladimir Salnikov (Ecole Normale Supérieure de Lyon, France)

The dynamics of the triple pendulum: various approaches to non-integrability

Summary: A multiple pendulum is a system of mass points with constraints given by second degree polynomial equations. In this work the free planar motion of it is studied as a test problem for comparison of various approaches to integrability. Special attention is given to topological reduction of the phase space of the system, giving the opportunity to show the non-integrability using numerical simulations. Analogous results are stated for the behavior of geodesics on the tori with special metric.

Tatiana Salnikova (Moscow State Lomonosov University, Russia)

Periodic solutions of one variant of the bounded three-dimensional three-body problem

Summary: A three-body problem is formulated as follows. Two points of equal masses, moving in the (x,y)-plane, circumscribe the elliptical orbits, which are symmetric relative to z-axis. The third point of a zero mass holds stuck on z-axis. Periodic solutions are studied for a small value of eccentricity in terms of variables “angle-action”. Eccentricity is considered to be a small parameter in the problem. An infinite multitude of families involving non-degenerated long-period solutions was proved to exist using Poincaré theorem. The solutions are accumulated in vicinity of separatrices obtained in a non-perturbed problem. This result is indicative of intricate chaotic movement of the system involved in the range under investigation.

Alexey Samokhin (Moscow State Technical University of Civil Aviation, Russia)

On deformations conserving a conservation law

Summary: In the case of a system of PDEs having a conservation law it is always possible to deform the system in such a way that it will still have the same conserved quantity. The effect of such a transform on the symmetry algebra of PDEs is studied.

Jan Sanders (Vrije Universiteit Amsterdam, Netherlands)

An addition formula for nilpotent normal forms

Summary: We offer an algorithm to determine the form of the normal form for a vector field with a nilpotent linear part, when the form of the normal form is known for each Jordan block of the linear part taken separately. The algorithm is based on the notion of transvectant, from classical invariant theory (joint work with J. Murdock, Ames).

Paolo M. Santini (Università di Roma “La Sapienza”, Italy)

The dispersionless Kadomtsev-Petviashvili equation: Cauchy problem, longtime behavior and wave breaking

Summary: We apply the spectral transform for pencils of multidimensional vector fields, recently developed by the authors, to the dispersionless Kadomtsev-Petviashvili equation, a nonlinear model in 2+1 dimensions describing small amplitude, two dimensional shallow water waves near the shore. In particular, we solve its Cauchy problem and clarify the spectral mechanisms underlying the breaking of any localized two dimensional initial condition. This is joint work with S.V. Manakov.

Gianfranco Sartori (Università di Padova, Italy)

No communication presented

David Saunders (Palacki University, Olomouc, Czech Republic)

Homogeneous Variational Systems

Summary: Homogeneous variational problems are defined on spaces of non-degenerate velocities; the homogeneity property implies that any reparametrization of an extremal is again an extremal. This approach to variational problems allows us to isolate the effect of changing the independent variables, and gives rise to some simplifications in the geometry. In this talk we describe a version of the variational bicomplex which is appropriate for homogeneous problems, and examine some of its properties.

Christian Scimiterna (Università di Roma Tre, Italy)

Multiscale expansion of the lattice potential KdV and of its symmetries on functions of infinite order

Summary: In this paper we present the discrete multiscale expansion of the lattice potential Korteweg-de Vries (lp-KdV) equation and of its symmetries on functions of infinite order of slow-varyness. To do so we introduce a formal expansion of the shift operator on many lattices holding at all orders. The lowest secularity condition from the expansion of the lp-KdV equation gives a nonlinear equation, depending on shifts of all orders, of the form of a local nonlinear Schrodinger equation (NLS) equation. By expanding the symmetry group parameter λ in ϵ , the perturbation parameter, we get that the lowest order expansion of all symmetries of the lp-KdV equation gives a point symmetry of the NLS-type equation, while the higher order one provides nonlocal equations.

Francesco Strazzullo (Utah State University, Logan, USA)

Darboux Integrable Hyperbolic PDE's in the Plane of Generic Type

Summary: I will present some results and examples of EDS methods used to obtain a new and complete classification of Darboux integrable hyperbolic PDE's in the plane of generic type (non Monge-Ampère).

Marco Antonio Teixeira (UNICAMP, Campinas, Brazil)

Singularities of non-smooth dynamical systems

Summary: We present a classification of typical singularities of non-smooth dynamical systems in the 3-dimensional case. Aspects of structural stability and generic bifurcation are discussed.

Piergiulio Tempesta (Scuola Normale Superiore, Pisa, Italy)

Formal groups, L-series and hyperfunctions

Summary: A new construction relating formal groups, Bernoulli-type polynomials and Dirichlet L-series is proposed. It generalizes the correspondence between the standard Bernoulli polynomials and the Riemann zeta function. By using the Fourier expansion of the Bernoulli-type polynomials, a generalization of the classical Lipschitz summation formula is also proposed. Related families of one-dimensional hyperfunctions are explicitly constructed. Work partially done in collaboration with S. Marmi.

Susanna Terracini (Università di Milano-Bicocca, Italy)

Singularities and collisions of generalized solutions to the N-body problem

Summary: The validity of Sundman-type asymptotic estimates for collision solutions is established for a wide class of dynamical systems with singular forces, including the classical N -body problems with Newtonian, quasi-homogeneous and logarithmic potentials. The solutions are meant in the generalized sense of Morse (locally -in space and time- minimal trajectories with respect to compactly supported variations) and their uniform limits. The analysis includes the extension of the Von Zeipel's Theorem and the proof of isolatedness of collisions. Estimates on the contribution of collisions to the Morse index will be discussed. Furthermore, such asymptotic analysis is applied to prove the absence of collisions for locally minimal trajectories and, therefore, existence of new periodic and almost-periodic solutions for the N -body problem which are equivariant under the action of an appropriate symmetry group.

Juergen Tolksdorf (Max Planck Institute, Leipzig, Germany)

Dirac Type First Order Differential Operators as a Natural Square Root of Gravity and Yang-Mills Gauge Theories

Summary: We discuss gauge theories in terms of twisted Grassmann bundles and Dirac type operators. For this we introduce a universal functional on the affine manifold of all Dirac type operators which is a natural generalization of the Einstein-Hilbert functional. This functional also covers gauge theories of

Yang-Mills type, with and without spontaneous symmetry breaking, in a way that is most similar to Einstein's theory of gravity. The presented geometrical description of gauge theories in terms of Dirac type operators may thus be considered as a unifying background of gravity and Yang-Mills gauge theories.

Enrico Valdinoci (Università di Roma "Tor Vergata", Italy)

Periodic and quasiperiodic motions in the many body planetary problem

Summary: We deal with the spatial planetary three-body problem (i.e., one "star" and two "planets", modelled by three massive points, interacting through gravity in a three dimensional space). Near the limiting stable solutions given by the two planets revolving around the star on Keplerian ellipses with small eccentricity and small non-zero mutual inclination, the system is proven to have two-dimensional, elliptic, quasi-periodic solutions, provided the masses of the planets are small enough compared to the mass of the star and provided the osculating Keplerian major semi-axes belong to a two-dimensional set of density close to one. We also find periodic orbits of longer and longer period which approach the above quasi-periodic solutions. The latter result is in fact a consequence of a general Birkhoff-Lewis type theorem which shows the existence of infinitely many periodic solutions, with larger and larger minimal period, accumulating onto elliptic invariant tori of Hamiltonian systems. The existence of elliptic quasi-periodic orbits for the planar N -body problem is also discussed.

Andre Vanderbauwhede (Ghent University, Belgium)

Continuation of doubly-symmetric solutions in reversible systems

Summary: In this lecture we will discuss some results on the continuation of so-called "doubly-symmetric solutions" in reversible systems with one or more first integrals. Such doubly-symmetric solutions will typically appear in one- or more-parameter families; in many cases they are automatically periodic. We first introduce the concept of a "quasi-submersive mapping" and obtain the main properties of such mappings. Next we introduce "normality conditions" under which a certain class of "constrained mappings" are quasi-submersive at their zeros; this then leads to some general continuation results for such zeros. The continuation problem for doubly-symmetric solutions can be reformulated in terms of such constrained mappings, and the normality condition can be specified in terms of an appropriate monodromy matrix. We illustrate the theoretical results with some numerical study of the continuation of the figure-eight and the supereight choreographies in the n -body problem. The work presented in this lecture was done in collaboration with F.J. Munoz-Almaraz (Valencia), E. Freire and J. Galan (Sevilla).

Pieter van der Kamp (La Trobe University, Melbourne, Australia)

Towards Global Classifications: a Diophantine Approach

Summary: I will present a global classification of approximately integrable 2-component evolution equations. Here approximately refers to the notion intro-

duced by Mikhailov and Novikov in what they called the Perturbative Symmetry Approach.

Andrea Venturelli (Université d'Avignon, France)

Globally Minimizing Parabolic Solutions for the Newtonian N-body Problem

Summary: We study the Newtonian N-body problem and we show the existence of a minimizing Parabolic solution, starting from a given configuration x_i and asymptotic to a given minimizing central configuration x_0 . The solution is obtained as limit of minimizers with mixed ends. It is a joint work with E. Maderna from the University of Montevideo.

Ferdinand Verhulst (University of Utrecht, Netherlands)

Emergence and break-up of invariant manifolds in a parametric PDE

Summary: Not provided

Raffaele Vitolo (Università del Salento, Lecce, Italy)

No communication presented

Renato Vitolo (Università di Camerino, Italy)

The Hopf-saddle-node bifurcation for fixed points of 3D-diffeomorphisms: a dynamical inventory

Summary: A numerical study of the Hopf-saddle-node (HSN) bifurcation for fixed points of diffeomorphisms is presented. The phase space is three-dimensional and we need three parameters to give an appropriate description. The exclusion of strong resonances implies that near the central singularity the diffeomorphisms can be approximated very well by the flow of a corresponding HSN vector field case. A model map is constructed that is on the one hand near the flow of a “versal” HSN vector field unfolding, and, on the other hand, designed “as generic as possible” as diffeomorphisms go. Lyapounov exponents and computation of normal behavior of invariant circles by Fourier analysis are used for a systematic exploration of bifurcation patterns and dynamical phenomena of the model map. The study is centered around a 1:5 resonance taking place on a frayed quasi-periodic bifurcation boundary in a suitable parameter plane. Several subordinate quasi-periodic bifurcations of codimension 1 and 2 of invariant circles and two-tori occur in the neighborhood, leading to the formation of strange repellers. Another phenomenon of interest are chaotic regions, as well as routes that lead to quasi-periodic Henon-like strange attractors. In particular, a quasi-periodic period doubling route is presented. This is joint work with H. Broer and C. Simo’.

Sebastian Walcher (RWTH Aachen, Germany)

The Michaelis-Menten equation and Murphy’s law

Summary: The quite well-known Michaelis-Menten equations for an enzyme-catalyzed biochemical reaction, and the quasi-steady state assumptions going

along with them, provide surprisingly many problems, since the standard approaches via singular perturbation theory do not work. Even worse, not only are the hypotheses of Tychonov's theorem not satisfied, but the conclusions also do not hold. In this talk we provide a complete discussion of these systems, using phase-plane analysis and some theory of monotone dynamical systems.

Pavel Winternitz (Université de Montreal, Canada)

Superintegrable systems in quantum mechanics

Summary: Superintegrable systems in classical and quantum mechanics are those which allow more integrals of motion than degrees of freedom. A large body of literature exists on such systems with a scalar potential and with integrals of motion that are second order polynomials in the momenta. Here we shall review some new results that go beyond this framework. Thus, we shall consider systems with spin allowing spin orbital interactions, and also scalar systems with higher order integrals of motion.

Irina Yehorchenko (Institute of Mathematics of the National Academy of Sciences, Kyiv, Ukraine)

Relative Invariants of Lie Algebras: Construction and Applications

Summary: We discuss methods for construction of relative differential invariants for Lie algebras that is relevant, in particular, for description of equations invariant under these algebras. We specifically consider examples of vector functions and group classification of the systems of equations for such vector functions.

Boris Zhilinskii (Université du Littoral, Dunquerque, France)

Generalization of Hamiltonian monodromy. Quantum manifestations.

Summary: New qualitative features which appear in the joint spectra of several mutually commuting quantum operators are interpreted in a way similar to quantum Hamiltonian monodromy. This leads to new notions in classical integrable systems and in corresponding quantum problems. I'll concentrate on "bidromy" and on its manifestations in quantum molecular systems.