small SPT2023 – Otranto, 4-10 June 2013

May 21, 2023

1 Talks & abstracts

1. **S. Bertrand** (Hawaii) *Representation of Racah algebra of higher rank*

   The Racah algebra appears in more than one field of Mathematical Physics, e.g. superintegrable systems on the hypersphere and the $6j$-symbols. In this presentation, we use a split basis to represent the rank-2 Racah algebra on a triangular domain. Some symmetries of the generators and properties of the algebra will be discussed. Joint work with Sarah Post.

   `https://arxiv.org/abs/2001.11195`

   `https://arxiv.org/abs/2206.01031`

   `https://arxiv.org/abs/1910.11446`

2. **M. Casati** (Ningbo), *Hamiltonian structures for nonabelian differential-difference systems*

   Integrable nonabelian systems are equations of motion in which the field variables take values in a noncommutative algebra, as a matrix one. In a series of papers with Jing Ping Wang (Nonlinearity 2021, CMP 2022) we have investigated the Hamiltonian structure and recursion operators for hierarchies of differential-difference integrable equations, providing a geometrical interpretation that helps to shed some light onto the structure on nonabelian Hamiltonian systems in general. I will present the notions of double multiplicative Poisson vertex algebra and of nonabelian functional polyvector fields, with the latter as the natural language to describe Hamiltonian systems in the familiar geometrical terms. As an application, I will discuss some results towards the classification of scalar Hamiltonian difference structures and present a list of such structures for the nonabelian generalization of well-known integrable systems such as Volterra, Toda and Kaup lattices.

3. **A.B. Cruzeiro** (Lisboa), *Stochastic variational principles on Lie groups*

   We derive variational methods for certain deterministic equations of motion corresponding to dissipative systems, ODE's (finite dimensions) or PDE's (infinite dimensional case) by deforming stochastically underlying Lagrangian paths and interpreting velocities in a generalized sense. Examples: Navier-Stokes, MHD. This is joint work with X. Chen and T.S. Ratiu.

4. **C. Chanu** (Aosta), *Dynamics of Hamiltonian systems on Riemannian covering of constant curvature*

   Some covering of constant curvature manifold appear implicitly in the literature related with superintegrable systems with a constant of the motion given as an arbitrary degree in the momenta polynomial, such as the Tremblay-Turbiner-Winternitz system. We introduce a family of Riemannian metrics depending on a rational parameter k and we discuss properties of Hamiltonian systems defined on their cotangent bundle, such as superintegrability and superseparability.

   `https://arxiv.org/pdf/2208.12690.pdf`

   `https://arxiv.org/abs/2001.08613`

5. **M. Dunajski** (Cambridge), *Elizabethan Vortices*

   Abelian vortices are solitons in two space dimensions, with finite core size. I will explain how radial solutions to the sinh-Gordon equation and the elliptic Tzitzeica equation can be interpreted as abelian vortices on certain surfaces of revolution. These surfaces have a conical excess angle at infinity (in a way which makes them similar to Elizabethan ruff collars, or certain green algae). They can not be embedded in the Euclidean 3-space but can be globally embedded in the hyperbolic space. I will show pictures of these embeddings, and discuss their existence which follows from the asymptotic analysis of a Painleve III ODE .

6. **F. Fasso'** (Padua), *Spectral analysis of a system of pendula hanging from a viscoelastic string and of their synchronization*

   The talk reports the study of the spectrum of the linearization of a hybrid (discrete-continuous) mechanical system formed by a number of identical pendula hanging from a (Kelvin-Voigt) viscoelastic string, with an analysis of the ensuing synchronization patterns in the pendula's motion. Joint work with S. Galasso and A. Ponno.

7. **E. Ferapontov** (Loughborough), *Linearisable Abel equations and the Gurevich-Pitaevski problem*

   Applying symmetry reduction to a class of SL(2, R)-invariant third-order ODEs, we obtain Abel equations whose general solution can be parametrised by hypergeometric functions. Particular case of this construction provides a general parametric solution to the Kudashev equation, an ODE arising in the Gurevich-Pitaevskii problem, thus giving the first term of a large time asymptotic expansion of its solution in the oscillatory (Whitham) zone. Based on joint work with Stanislav Opanasenko.

   `http://arxiv.org/abs/2202.07512`

8. **M. Gorgone** (Messina), *Reduction of nonlinear PDEs to homogeneous and autonomous form*

   Using Lie symmetries of differential equations, we exploit necessary and /or sufficient conditions allowing us to reduce, through an invertible point transformation, nonlinear PDEs to homogeneous and autonomous form. Some examples of physical interest are presented.

   `https://arxiv.org/pdf/2108.00042.pdf`

   `https://arxiv.org/pdf/2107.14671.pdf`

9. **G. Gubbiotti** (Milano), *Detecting maximal superintegrability of Aristotelic equations of motion with perturbation techniques*

   We present a method to determine if a system of Aristotelic equations of motion, i.e. a system of M first order differential equations admits M-1 first integrals. This method is analog to the one for Newtonian/Lagrangian/Hamiltonian equations presented in [G. Gubbiotti and D. Latini, J. Phys. A., 2018]. In particular we present some new results on Lotka-Volterra and related systems.

10. **Y. Kodama** (Shandong & Ohio State), *Minimal completion and the Whitham theory for KP solitons*

    I will discuss initial value problem of the KP equation based on the Whitham theory of averaging. We consider certain class of initial data close to some exact solitons, and show that the solution converges to the exact one as a result of 'minimal' completion of the corresponding chord diagram of permutation.

11. **B. Konopelchenko** (Salento), *On blowups of vorticity for the homogeneous Euler equation*

    Blowups of vorticity for the three- and two-dimensional homogeneous Euler equations are discussed. The existence of blowups of different degrees is demonstrated.

12. **R. Kozlov** (NSE Bergen), *Lie point symmetries of stochastic differential equations and their applications*

    Lie point symmetries of Ito stochastic differential equations (SDEs) are considered. They correspond to Lie group transformations of the independent variable (time), dependent variables and the Brownian motion. These transformations preserve the differential form of the SDEs and properties of the Brownian motion. There are provided some properties of the SDEs symmetries: the symmetries form a Lie algebra, there are relations between the symmetries and the first integrals, the symmetries of SDEs are related to symmetries of the Kolmogorov forward equation (also known as the Fokker-Planck equation) and the Kolmogorov backward equation. The symmetries can be used integrate SDEs with the help of quadratures and to construct Lie symmetry group classifications.

13. **B. Kruglikov** (Tromso) *Higher dimensional dispersionless integrable systems*

    Dispersionless integrability in dimensions 3 and 4 is quite well understood using the tools of differential geometry, twistor theory, and group theoretic approach. (Dimension 2 is very special, and is also understood.) In particular, hydrodynamic integrability and the existence of dispersionless Lax pairs is related to Einstein-Weyl conformal structures in 3D and self-dual conformal gravity in 4D, which was made explicit for classes of integrable systems in a collaboration with Eugene Ferapontov. Higher dimensional systems are much less understood. Only few examples of such systems have been investigated and almost no general theory exists. What is known, however, due to my joint work with David Calderbank, is that there are no nondegenerate second order dispersionless integrable PDE in dimensions higher than 4. But there are integrable degenerate examples in all higher dimensions. This talk is devoted to the geometry behind integrability in higher dimensions, and is based on a current collaboration with Omid Makhmali.

14. **O. Kubu** (CTU Praha) *Quadratically integrable systems in magnetic field of nonstandard types*

    As shown by Marchesiello and Snobl [J. Phys. A 2022], quadratic integrable system with magnetic fields may admit a more general form of integrals of motion that is no longer connected to separability. We summarize results on integrability and superintegrability of this type of systems obtained in collaboration with L. Snobl, A. Marchesiello, and M. Hoque.

    `https://arxiv.org/abs/2206.15305`

    `https://arxiv.org/abs/2212.05338`

15. **A. Marchesiello** (CTU Praha), *Symmetric resonances and related bifurcations*

    We consider resonant normal forms corresponding to a wide class of Hamiltonian systems. In particular, we are interested in systems that exhibit symmetries, as systems symmetric under reflection with respect to one or more coordinates axes or rotationally symmetric systems. We study the bifurcations related to the resonances in their own right, not restricted to natural Hamiltonian systems where H=T+V would consist of kinetic and (positional) potential energy.

    https://arxiv.org/pdf/2005.09686.pdf

    https://arxiv.org/pdf/1807.09453.pdf

16. **F. Mokhtari** (VU Amsterdam), *The linear Lie algebraic structure of colored network dynamics*

    We are interested in the Lie algebraic structure of colored network dynamical systems. In the classification of the normal form of a colored network vector field, following the semigroup(oid) approach in [3], one would like to be able to say something about the structure of the Lie algebra of the linear colored networks. We describe in [2] a concrete algorithm (cf. [1]) that gives us the block decomposition for the Lie algebra of the linear part ($\mathtt{net}_{(C,N)}$) as a block matrix with $B = N - C$. We show that for N-dimensional vector fields with C colors (different functions describing different types of cells in the network) this Lie algebra $\mathtt{net}_{(C,N)}$ is isomorphic to the semidirect sum of a semisimple part, consisting of two simple components $\mathtt{sl}_C$ and $\mathtt{sl}_B$, which we write as a block-matrix, and a solvable part, consisting of two elements representing the identity $c \simeq \mathtt{gl}_C$ and $b \simeq \mathtt{gl}_B$, and an abelian algebra $a \simeq \mathtt{Gr}_{C,N}$, the Grassmannian, consisting of the C-dimensional subspaces of R^n. This is joint work with J. Sanders.

    https://www.tandfonline.com/doi/full/10.1080/14689367.2016.1235136

    https://arxiv.org/abs/2109.11419

    https://arxiv.org/abs/1209.3209

17. **C. Muriel** (Cadiz), *Solvable structures and generalizations*

    We review the concept of soluble structure, as well as related notions that have appeared in recent literature. Applications to integrability problems of vector field distributions are presented. New procedures for integrating differential equations are derived as special cases of the general framework.

18. **F. Oliveri** (Messina) *Computer assisted derivation of optimal systems of Lie subalgebras*

    Lie groups of point symmetries of partial differential equations constitute a fundamental tool for constructing group-invariant solutions. The number of subgroups is potentially infinite and so the number of group-invariant solutions. An important goal is a classification in order to have an optimal system of inequivalent group-invariant solutions from which all other solutions can be derived by action of the group itself. In turn, a classification of inequivalent subgroups induces a classification of inequivalent Lie subalgebras, and vice versa. A general method for classifying the Lie subalgebras of a finite dimensional Lie algebra relies on the use of inner automorphisms. A new improved version of the *SymboLie* package, written in Wolfram Mathematica, that can automatically determine optimal systems of Lie subalgebras of a generic finite-dimensional Lie algebra, is described. Some open problems originated by the use of the program will be also discussed.

19. **P.J. Olver** (Minneapolis), *Two New Developments for Noether's Two Theorems*

    Noether's First Theorem relates strictly invariant variational problems and conservation laws of their Euler–Lagrange equations. The Noether correspondence was extended by her student Bessel-Hagen to divergence invariant variational problems. In the first part of this talk, I highlight the role of Lie algebra cohomology in the classification of the latter, and conclude with some provocative remarks on the role of invariant variational problems in fundamental physics.

    In the second part, I start by recalling the two well-known classes of partial differential equations that admit infinite hierarchies of higher order generalized symmetries: 1) linear and linearizable systems that admit a nontrivial point symmetry group; 2) integrable nonlinear equations such as Korteweg–de Vries, nonlinear Schrödinger, and Burgers'. I will then introduce a new general class: 3) underdetermined systems of partial differential equations that admit an infinite-dimensional symmetry algebra depending on one or more arbitrary functions of the independent variables. An important subclass of the latter are the underdetermined Euler–Lagrange equations arising from a variational principle that admits an infinite-dimensional variational symmetry algebra depending on one or more arbitrary functions of the independent variables. According to Noether's Second Theorem, the associated Euler–Lagrange equations satisfy Noether dependencies; examples include general relativity, electromagnetism, and parameter-independent variational principles.

20. **J. Palacián** (Navarra), *Bifurcations in the Riemann ellipsoids*

    We study steady motions of an ideal incompressible homogeneous self-gravitating fluid mass. The ones that retain an ellipsoidal shape are called Riemann ellipsoids. We present an approach based on Hamiltonian normal forms to prove the existence of pitchfork, centre-saddle and Hamiltonian-Hopf bifurcations.

21. **G. Pucacco** (Rome – Tor Vergata), *Normal forms for Laplace-like resonances*

    We describe the generalisation of the de Sitter equilibria in multi-resonant gravitational 1+3 body systems in the case of first-order resonances. The prototype of these systems are the Galilean satellites of Jupiter. We show that, under certain conditions on the proximity parameter to the exact resonance, additional equilibria with large forced eccentricities are possible. The analysis of stability provides hints for the structure of multi-resonant chains in exo-planetary systems.

22. **G. Rastelli** (Torino), *Separation of variables and superintegrability on Riemannian coverings*

    We introduce Staeckel separable coordinates on covering manifolds of certain constant-curvature Riemannian manifolds. These coverings are connected with superintegrable systems such as the Tremblay-Turbiner-Winternitz system. We study for the first time multiseparability and superintegrability of natural Hamiltonians on these manifolds and see how these properties depend on a rational parameter.

23. **M.A. Rodríguez** (Madrid), *Ito equation: a search for symmetries and exact solutions*

    Ito equation describes the evolution of a stochastic variable which depends on a time variable and a Wiener process. The equation (which can be written in differential form) involves two functions (in the simple scalar case, noise and drift in some applications). Following the approach of Gaeta and coworkers, I will discuss in this talk how the Lie theory of symmetries of differential equations can be adapted to stochastic differential equations. I will present a short introduction to this topic and some recent results on the classification of Ito equations admitting a symmetry. I will also discuss the Kozlov method to compute exact solutions of Ito equations allowing a symmetry.

24. **A. Ruiz Serván** (Cadiz), *Exact solutions to position-dependent mass damped oscillators via variational λ-symmetries*

    In this talk we study a wide family of position-dependent mass damped oscillators affected by an external potential. A Lagrangian formulation for the corresponding problem is first introduced. Since the corresponding variational problem lacks variational symmetries, then the variational λ-symmetry method is applied in order to find exact solutions. Variational λ-symmetries are determined for a family of potential functions, leading to a one-parameter family of exact solutions to the corresponding motion equations.

25. **H. Saberbaghi** (GSSI), *A new look at the theory of point interactions*

    A careful look at the entire family of self-adjoint extensions of the Laplacian known as quantum point interaction Hamiltonians shows that the great majority of them do not become either singular or trivial when the positions of two or more scattering centers tend to coincide. I will summarize some properties of these sub-family of extensions and try to clarify the renormalisation mechanism which makes them regular and physically relevant. I will show how the use of these Hamiltonians in the Born-Oppenheimer approximation of the three particle dynamics avoids the unboundedness problem and predicts a correct Efimov spectrum at low energy. This is a joint work with Rodolfo Figari and Alessandro Teta.

26. **P. Santini** (Rome – Sapienza) *Towards an analytic theory of periodic 1+1 and 2+1 dimensional anomalous waves in nature*

    Anomalous (rogue) waves (AWs) are transient waves of anomalously large amplitude compared to the average, appearing, apparently from nowhere, in several physical contexts like deep water, nonlinear optics, and Bose-Einstein condensates. Modulation instability and nonlinearity are the main causes of the appearance of AWs, and the integrable focusing cubic nonlinear Schrödinger equation, relevant in the description of the amplitude modulation of monochromatic waves in weakly nonlinear media, is the basic model in the description of this phenomenon in 1+1 dimensions. The analytic theory of periodic AWs has been recently constructed (with P.G. Grinevich), and tested in the nonlinear optics of a photorefractive crystal. Also a perturbation theory of AWs has been constructed, allowing one to study the order one effects of small perturbations of the NLS equation on the AW dynamics (with P.G. Grinevich and F. Coppini). This theory is in the process of being extended to 2+1 dimensions (with P.G. Grinevich and F. Coppini), using as basic model the focusing Davey-Stewartson 2 equation, an integrable 2+1 dimensional generalization of NLS. In this lecture we summarize all these results.

27. **L. Snobl** (CTU Praha), *Pairs of commuting quadratic elements in the universal enveloping algebra of Euclidean algebra and integrals of motion*

    Motivated by the consideration of integrable systems in three spatial dimensions in Euclidean space with integrals quadratic in the momenta we classify three-dimensional Abelian subalgebras of quadratic elements in the universal enveloping algebra of the Euclidean algebra under the assumption that the Casimir invariant $\mathbf{p} \cdot \ell$ vanishes in the relevant representation. We show explicit examples demonstrating that in the presence of magnetic field, i.e. terms linear in the momenta in the Hamiltonian, this classification allows for pairs of commuting integrals whose leading order terms cannot be written in the famous classical form of [Makarov, Smorodinsky, Valiev and Winternitz, Il Nuovo Cimento A 10 (1967) 106184]. Some of these models find direct physical application, e.g. in description of helical undulators in magnetic fields.

    `https://arxiv.org/abs/2206.15305`

28. **P. Tempesta** (UCM Madrid & ICMAT), *Generalized Nijenhuis geometry: A new family of tensor fields and integrability*

    We propose a new, infinite family of tensor fields, whose first representatives are the classical Nijenhuis and Haantjes tensors. We prove that the vanishing of a suitable higher-level Haantjes torsion is a sufficient condition for the integrability of the eigen-distributions of an operator field on a differentiable manifold. This new condition, which does not require the explicit knowledge of the spectral properties of the considered operator, generalizes the celebrated Haantjes theorem, because it provides us with an effective integrability criterion applicable to the generic case of non-Nijenhuis and non-Haantjes tensors. We also introduce the notion of algebras of generalized Nijenhuis tensors, consisting of operators with a vanishing higher-level torsion. For these algebras there exists local charts where all the operators can be block-diagonalized simultaneously. This result represents a contribution to the Courant problem of determining normal forms of systems of PDEs. Multivalued generalizations of the previous construction, in particular of the Frolicher-Nijenhuis bracket, are also proposed. Work in collaboration with D. Reyes and G. Tondo.

    `https://link.springer.com/article/10.1007/s00220-021-04233-5`

    `https://arxiv.org/abs/2209.12716`

    `https://arxiv.org/abs/2205.09432`

29. **G. Tondo** (Trieste), *Symplectic-Haantjes manifolds: a new route to integrable Hamiltonian systems*

    A tensorial approach to the theory of classical Hamiltonian integrable systems is proposed, based on the geometry of Haantjes operators. They represent a natural generalization of the well-known class of Nijenhuis operators, which has a fundamental role in many geometric contexts. We introduce the family of symplectic-Haantjes manifolds as a natural setting where the notion of integrability can be formulated. We prove that the existence of symplectic-Haantjes manifolds is a necessary and sufficient condition for a Hamiltonian system to be integrable in the Liouville-Arnold sense. The theory of classical Hamiltonian systems admitting separating variables can also be formulated in the context of our new geometric structures. Work in collaboration with D. Reyes and P. Tempesta.

    `https://link.springer.com/article/10.1007/s10231-021-01107-4`

    `https://arxiv.org/abs/1710.04522`

    `https://arxiv.org/abs/2012.09819`

30. **C. Tronci** (Surrey), *Fluid models of mixed quantum-classical dynamics*

    In order to overcome the computational challenges of fully quantum simulations, mixed quantum-classical (MQC) models have been proposed in physics and chemistry, so that parts of a quantum system are treated classically while the remainder is left fully quantum. However, current MQC descriptions typically suffer from long-standing consistency issues, and, in some cases, invalidate Heisenberg's uncertainty principle. A step forward was recently provided by our phase-space Hamiltonian description of MQC dynamics, which blends van Hove's work on symplectic geometry with Koopman's Hilbert-space formulation of classical dynamics. While this phase-space model succeeds in satisfying important consistency properties, one is motivated to devise suitable fluid closures thereby alleviating the curse of dimensionality in phase-space treatments. Also, in computational chemistry, MQC fluid models represent a convenient mesoscopic description of the coupling between quantum solute molecules and the surrounding fluid solvent in solvation dynamics. This talk shows how suitable fluid closures can be devised by operating directly on the underlying phase-space variational principle of the original model. In particular, we will see how Nambu brackets appear to play a peculiar role in realizing quantum-classical coupling. As a result of its Hamiltonian structure, the new MQC fluid model possesses infinite families of Casimir invariants that may be used for Liapunov stability studies. Joint work with François Gay-Balmaz (CNRS/ENS Paris)

    `https://arxiv.org/abs/2112.12144`

31. **V. Vassilev** (BAS Sofia), *Exact solutions to the cubic-quintic complex Ginzburg-Landau equation*

Various phenomena in the theory of phase transitions, nonlinear optics (wave propagation in optical fibers), laser physics, fluid mechanics, and many other branches of physics, are modelled on the ground of the so-called cubic-quintic complex Ginzburg-Landau equation (CQCGLE). Actually, it is a generalization of the standard nonlinear Schroedinger equation. The CQCGLE captures dispersive and nonlinear effects. In this talk, I will present, in analytic form, new exact solutions to this nonlinear equation, which represent solitary waves of kink type. These solutions are obtained in the following way. First, adopting a suitable ansatz, the considered equation is reduced to a system of two second-order nonlinear ordinary differential equations for the amplitude and phase functions (speaking in terms of nonlinear optics). Then, two differential constraints are added and their compatibility with the regarded system is studied. A wide variety of compatible cases is obtained when the amplitude function is a solution to a Bernoulli equation of constant coefficients and the derivative of the phase function is equal to a second order polynomial of the amplitude function. Finally, using the solutions of the respective Bernoulli equation, which can be readily obtained in closed form, one constructs solutions to the regarded complex Ginzburg-Landau equation. As a by-product, exact solutions of a family of Liénard equations are also obtained and will be presented.

32. **P. Vergallo** (Milano) *Hamiltonian structures for systems in Jordan block form*

We consider evolutionary quasilinear systems of first order $u^i_t = V^i_j(u)u^j_x$ $(i = 1, 2, \ldots n)$. In the case that $V^i_j(u)$ is a diagonal matrix (in particular, with distinct eigenvalues), there is a huge literature investigating the integrability, the solutions (by means of the generalized hodograph method) and the Hamiltonian structures with first order homogeneous Hamiltonian operators. Some recent developments have been achieved by E.V. Ferapontov, M.V. Pavlov and L. Xue in the case when the matrix $V^i_j(u)$ is block-diagonal with several upper-triangular blocks. We refer to such systems as being of Jordan block (Toeplitz block) type. In the present poster, we wonder when a quasilinear system in Jordan block form is Hamiltonian, with first order homogeneous operators. Surprisingly, we show that for such systems linear degeneracy is a necessary condition to be Hamiltonian. Finally, we investigate the Hamiltonian structure of El's kinetic equations for soliton gases after delta-functional reductions. We find that the above mentioned structure is obtained for separable 2-soliton interaction kernels. Based on a joint work with E.V. Ferapontov.

`https://arxiv.org/abs/2212.01413`

33. **F. Verhulst** (Utrecht), *The emergence of tori*

    A remarkable aspect of the system Sprott A and a few generalisations is the observed presence of families of invariant tori, known earlier in conservative systems. We can link the tori bifurcation phenomenon to time-reversal and canards. For more general isolated tori in dissipative systems we can develop an integral iteration scheme based on contraction and quasi-periodic secularity conditions. The technique leads to conditions for the presence of tori and possibly other invariant manifolds. The general idea can be illustrated by an example.

34. **A.P. Veselov** (Loughborough), *Symmetry and shadow dynamics*

    I will discuss the integrability in the dynamics over dual numbers, for which Valentin Ovsienko recently coined the term shadow dynamics. A particular example will be the discrete dynamics determined by the modular group acting as the symmetry of the Cayley cubic surface.

35. **R. Vitolo** (Salento), *Plucker embedding and bi-Hamiltonian systems*

    In recent works, a correspondence between certain classes of Hamiltonian operators and algebraic varieties in the Plucker embedding of the space of field variables was established. In this paper, we carry the correspondence a step further. Namely, for bi-Hamiltonian systems that have the bi-Hamiltonian pair of the form (P, Q + R), where P, Q are compatible first order homogeneous Hamiltonian operators and R is a second-order homogeneous Hamiltonian operator that is compatible with both P and Q, we find the projective varieties that correspond with P, Q and R. Examples of such systems are a dispersive water waves system by Kaup-Broer, the AKNS system, a two-component Camassa-Holm system and others. Strong indications that this result will hold for systems defined by P, Q, R where R is third-order will be presented as well. This is joint work with P. Lorenzoni.

36. **S. Walcher** (RWTH Aachen), *Invariant sets and reductions of the Michaelis-Menten system*

    The Michaelis-Menten reaction network is a classical model of an enzyme-catalyzed reaction. While it has been under investigation for more than a hundred years, its features are still not completely understood, both from a mathematical and a biochemical perspective. This talk will contain an account of some known features, open questions and new approaches.

37. **P. Yanguas** (Navarra), *Invariant 4-tori in the co-orbital motion of 3 bodies*

    A qualitative explanation of the co-orbital motion of two small moons orbiting a planet is presented. The system is modelled as a planar three-body problem whose Hamiltonian is expanded as a perturbation of two uncoupled Kepler problems. A combination of averaging, normal form, symplectic scaling, Hamiltonian reduction theories and the application of a KAM theorem for high-order degenerate systems allows us to establish the existence of quasi-periodic motions and KAM 4-tori related to the co-orbital motion of the moons.

    `https://academica-e.unavarra.es/handle/2454/32505`

38. **J.C. Zambrini** (Lisboa), *Schroedinger's problem and its stochastic dynamical solution*

    "Schroedinger's problem" is a recent name for a very old observation of Erwin Schroedinger (1931) about the mysterious probabilistic content of quantum mechanics.We shall summarize it, together with its modern geometrico-mechanical solution. Its original formulation was re-discovered and used , in the last decade, by the Mass Transportation community and regarded, there, as a fundamental problem between those of Monge (1781) and Kantorovich (1942).

2 Workshop program (tentative)

Monday 5			
09:15-09:45 09:45-10:00 10:00-10:30	*Registration* *Opening* Konopelchenko	16:00-16:20 16:20-16:40 16:40-17:10	Muriel Palacian Snobl
10:30-11:30	*Coffee break*	17:10-18:00	*Coffee break*
11:30-12:00 12:00-12:30	Marchesiello Verhulst	18:00-18:30 18:30-19:00	Kozlov Veselov
Tuesday 6			
 09:30-10:00 10:00-10:20 10:20-10:40	 Cruzeiro Gorgone Yanguas	16:00-16:15 16:15-16:30 16:30-16:45 16:45-17:15	Bertrand Kubu Vergallo Gubbiotti
10:40-11:30	*Coffee break*	17:15-18:00	*Coffee break*
11:30-12:30	Olver	18:00-18:30 18:30-19:00	Zambrini Santini
Thursday 8			
 09:30-10:00 10:00-10:30	 Tronci Pucacco	16:00-16:20 16:20-16:40 16:40-17:00	Tondo Rodriguez Saberbaghi
10:30-11:20	*Coffee break*	17:00-17:45	*Coffee break*
11:20-11:40 11:40-12:00 12:00-12:30	Ruiz Rastelli Dunajski	17:45-18:15 18:15-18:45	Kruglikov Walcher
		21:15-21:45	Winternitz memorial lecture (Snobl)
Friday 9			
 09:30-10:00 10:00-10:30	 Vitolo Oliveri	16:00-16:20 16:20-16:40 16:40-17:00	Casati Vassilev Mokhtari
10:30-11:20	*Coffee break*	17:00-17:45	*Coffee break*
11:20-11:40 11:40-12:00 12:00-12:30	Chanu Tempesta Fasso'	17:45-18:15 18:15-18:45 18:45-19:00	Ferapontov Kodama *Closing*
		19:00-20:00	*Farewell (party)*