Abstract booklet for SPT 2004

Cala Gonone, 30 May – 6 June 2004

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Mark J. Ablowitz

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Chazy-Darboux-Halphen Systems

At the turn of the last century Painlevé and his school obtained a class of ordinary differential equations in the complex (time) plane which have no movable branch points. The second order ODEs they found include elliptic functions and the now well known six Painlevé equations. Chazy (1909)studied third order ODEs and found examples whose solutions are analytic inside/outside a circle which consists of a natural boundary of singular points. In this lecture we will show that the general solution can be written in terms of modular functions and we will discuss connections between the Chazy equation and a system of equations studied by Darboux-Halphen (1878) a generalized 9th order system, the generalized Chazy equation and the self-dual Yang-Mills equations. Remarkably the Chazy equation will be also shown to correspond to a system of equations studied by Ramanujan in 1916!

Ivan Andronov

St.Petersburg University
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Zero range potentials as generalized models in structural acoustics

Vibrations of thin-walled mechanical constructions are often described by accepting approximate models that consider only some types of deformations. This essentially simplifies the analysis excluding the study of wave processes in the interior of elastic elements. Joints, cracks, reinforcements, etc. are described in the frame of these approximate models by fixing so-called contact conditions. These conditions can be treated as second level boundary conditions (boundary conditions inside usual boundary conditions fixed on the surface of elastic elements). Classical contact conditions allow point models of cracks, joints, attached masses and momentums, and other inhomogeneities to be introduced. However for practical applications accuracy of such models can appear insufficient. Some examples when next order terms are significant are to be presented.

Technique of selfadjoint operators perturbations in the form of zero-range potentials allows classical point models to be generalized and smaller order corrections to be included. The simplicity of solution construction remains the same as in classical models. The standard procedure of the generalized point models construction and many generalized point models are described in [I.V.Andronov Generalized point models in structural mechanics World Scientific 2002]. Some other models are to be presented.

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Ceni Babaoglu

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Wave propagation in an elastic medium: GDS equations

The problem of (2+1) (two spatial and one temporal) dimensional wave propagation in a bulk medium composed of an elastic material with couple stresses is considered. The aim is to derive (2+1) nonlinear model equations for the description of elastic waves in the far field. Using a multi-scale expansion of quasi-monochromatic wave solutions, it is shown that the modulation of waves is governed by a system of three nonlinear evolution equations. These equations involve amplitudes of a short transverse wave, a long transverse wave and a long longitudinal wave, and will be called the "generalized Davey-Stewartson equations". Under some restrictions on parameter values, the generalized Davey-Stewartson equations reduce to the Davey-Stewartson and to the nonlinear Schrödinger equations. Finally, some special solutions involving sech-tanh-tanh and tanh-tanh-tanh type solitary wave solutions are presented.

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Taoufik Bakri

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Parametric Excitation in Nonlinear dynamics

(Abstract not provided)

Vivina Barutello
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Periodic solutions for the equivariant 3-body problem

(Abstract not provided)

Sergio Benenti
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The geometrical theory of the separation of variables in the Hamilton-Jacobi equation

Basic notion of symplectic geometry on cotangent bundles and of Riemannian geometry. The intrinsic characterizations of the separation in the HJ equations. Applications.

Sergey Bolotin
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Shadowing chains of collision orbits for the 3 body problem

We consider almost collision solutions of the elliptic restricted 3-body problem with masses 1 and ϵ. When $\epsilon \to 0$ such solutions degenerate into chains of collision orbits of the Kepler problem. Poincaré named them periodic solutions of the second kind. By using Levi-Civita regularization, constructing almost collision solutions is reduced to shadowing sequences of homoclinic orbits of the regularized Hamiltonian system. We use variational methods to prove the existence of different types of shadowing orbits for small $\epsilon > 0$.

Maria Santos Bruzón
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Similarity Reductions of an Optical Model

(Abstract not provided)

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Alexander Burov
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Restricted problems in the Rigid Body Dynamics

(Abstract not provided)

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Francesco Calogero
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New solvable dynamical systems

Some new, presumably interesting, solvable dynamical systems will be presented.

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N-body systems with vanishing total mass

N-body systems with vanishing total mass naturally appear in the study of choreographies: in such solutions of Newton's equations, the bodies chase each other around the same curve, with the same phase shift between consecutive bodies. The first known choreography was found numerically by C. Moore in 1993, and its existence was proved in 1999 by A. Chenciner and R. Montgomery. Since this discovery, lots of choreographies have been found numerically (C. Simó, G. Gerver), and analytically (A. Venturelli, D. Ferrario, S. Terracini, K.-C. Chen), particularly thanks to a result due to C. Marchal. All these solutions require equal masses, and, for the newtonian potential in $1/r$ (r is the distance between two bodies), the only known fact about eventual choreographies with distinct masses is that they do not exist for $N \leq 5$.

As a consolation, for the logarithmic potential, the absence of absolute equilibria with vanishing total mass entails that, for any choreography, the masses

must be equal.
Besides this property of absolute equilibria, strange phenomena can occur with vanishing total mass for any potential. They are the result of the existence of a time-dependent first integral λ which takes the place of the center of inertia and is invariant under translation. Among those phenomena:
- There is no periodic orbit for $N = 2$.
- On a line, the 3-body problem is integrable when the total momentum vanishes.
- In the 3-body problem, if at some time λ and $\dot{\lambda}$ vanish, the motion is homographic (the ratios between the mutual distances are constant).
- The 4-body central configurations, i. e. the configurations which generate a homothetic collapse onto a fixed point, are the critical points of a 2-variable function.

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Luca Degiovanni
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New examples of trihamiltonian structures linking different Lenard chain

In literature there are many examples of bihamiltonian systems whose first integrals are organized in several distinct Lenard chains. Moreover, several cases in which one, or more, addictional structures link together the hamiltonias belonging to the same Lenard chain are well known. Recently a class of bihamiltonian system endowed with a third compatible Poisson structure establishing a recurrence relation between different Lenard chain has been presented. These systems are particulary interesting because they admit a Lax representation with a spectral parameter and, surprisingly, the polynomial defining the related spectral curve is a simultaneous Casimir function ot two Poisson pencils constructed with the three Poisson structures. In this talk, after a review of the general results on trihamiltonian stuctures coupling different Lenard chains, some new examples of these kind of systems are exhibited. In particular some extension of separable systems in the plane and of the Toda lattice are presented.

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Florin Diacu
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Saari's Conjecture in the Collinear Case

In 1970 Don Saari conjectured that the only solutions of the Newtonian n-body problem that have constant moment of inertia are the relative equilibria.

In spite of many attempts to solve it, the problem is still open. We prove the conjecture in the collinear case for any potential that involves only the mutual distances. In the case of homogeneous potentials, we show that the only collinear and non-zero angular momentum solutions are homographic motions with central configurations. This work, in collaboration with Ernesto Perez-Chavela and Manuele Santoprete, will be published in the Transactions of the AMS.

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Gregorio Falqui

Sissa
via Beirut 2/4, 34014 Trieste Italy

Gaudin Models and Bending Flows:
(bi)-Hamiltonian Properties and Separation of Variables

We discuss the bihamiltonian formulation of the (rational XXX) Gaudin models of spin-spin interaction, generalized to the case of sl(r)-valued spins. In particular, we focus on the homogeneous models. We find a pencil of Poisson brackets that recursively define a complete set of integrals of the motion, alternative to the set of integrals associated with the 'standard' Lax representation of the Gaudin model. These integrals, in the case of su(2), coincide wih the Hamiltonians of the 'bending flows' in the moduli space of polygons in Euclidean space introduced by Kapovich and Millson. By means of a such a bi-Hamiltonian structure, we find a new set of Separation Coordinates. This is a joint work with F. Musso (University of Rome III).

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Francesco Fassò

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Period-energy relation for quasi-hamiltonian systems

We investigate the validity of the period-energy relation (and of its analogous for quasi-periodic systems, that is, the existence of action variables) for vector fields which are Hamiltonian with respect to a non-closed nondegenerate two-forms. This complements old investigations of the case of a closed but degenerate two-form. The idea is trying to understand which individual geometric properties are responsible for the properties of standard Hamiltonian systems.

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Mark Fels

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A co-chain map for the G-invariant deRham complex

We example conditions on a Lie group action on a manifold such that a canonical cochain map exist from the G-invariant deRham complex to the deRham complex on the quotient.

Frédéric Faure

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(Maison des Magisteres Jean Perrin, CNRS)
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Scars in quantum chaos

We consider a linear automorphism on the torus which is a very simple example of hyperbolic dynamics. We consider a quantized version of version of it, which is therefore a very simple model for the study of quantum chaos. For this very particular quantum dynamics which posses many symmetries, we describe the construction of eigen-states which, in the semi-classical limit $h \to 0$, concentrate themselves partially on a periodic orbit (choosen a priori). More precisely, the associated semi-classical measure is $1/2$ the Liouville measure on phase space plus $1/2$ the Dirac measure on this periodic orbit. It is known (Schnirelman theorem) that for almost all eigenstates-sequence, the semi-classical measure is the Liouville measure. The examples we propose are therefore exceptions to this general rule. We will give an interpretation of these phenomena in terms of collective constructive interferences between periodics orbits, and will discuss the generic case of a non linear hyperbolic symplectomorphism on the torus.

Davide Ferrario

Università di Milano Bicocca
Milano, Italy

Some symmetric periodic orbits in the planar n-body problem

Symmetries and variational methods have been recently extensively used in the problem of finding periodic solutions of the Newtonian n-body problem. It is of some interest to classify all the symmetry groups yielding non-homographic solutions and to describe the properties of the occurring local minima. I would like to present also some issues and related problems, originated by some numerical experiments and known results.

Toshiaki Fujiwara

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Synchronised Similar Triangles
for Three-Body Orbit with Zero Angular Momentum

Geometrical properties of the orbit of three bodies with zero angular momentum are investigated.

If the moment of inertia is also constant along the orbit, the triangle whose vertexes are the positions of the bodies and the triangle whose perimeters are the momenta of the bodies are always similar ("Synchronised similar triangles"). This similarity yields kinematic equalities connecting mutual distances and magnitude of momenta. Moreover, under homogeneous potentials, we find a new constant involving momenta along the orbit. The three-body figure-eight solution under a non-Newtonian strong attractive force, whose strength is inversely proportional to the third power of mutual distances, has these properties.

For orbits with zero angular momentum and non-constant moment of inertia, we introduce the scaled variables, positions divided by square root of the moment of inertia and momenta derived from these scaled positions. Then the similarity and the kinematic equalities holds for the scaled variables. Using this similarity, we give a short proof of "infinitely many syzygies (eclipse)" for the three-body orbit with zero angular momentum.

Rustem Garifullin

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The asymptotic solution of the autoresonance problem

We study a nonlinear ordinary differential equation with forced oscillations:

$$\frac{d^2u}{dt^2} + F(u) = \varepsilon f \cos\left(\frac{\Phi(\tau)}{\varepsilon}\right), \quad t > 0,\ 0 < \varepsilon \ll 1,\ \tau = \varepsilon t.$$

Where $\Phi(\tau) = \tau + \tau^3\phi(\tau)$, $F(u) = u + au^2 + bu^3 + u^4 g(u),\ a^2 + b^2 \neq 0,\ a, b, f -$ constant; the functions $\phi(\tau), g(u)$ are smooth.

We are looking for solutions which start from a small neighborhood of the stable equilibrium point. For such solutions we construct asymptotics as $\varepsilon \to 0$

for long period $t \leq O(\varepsilon^{-1})$. We are interested in the solutions growing up to $O(1)$ at $t = O(\varepsilon^{-1})$. The autoresonance phenomenon is described by these solutions.

The main result of the work is constructing and justifying the two-parameter family of asymptotic solutions.

Guido Gentile
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Periodic solutions for zero-mass nonlinear wave equations

In a celebrated paper by Craig and Wayne existence of periodic solutions has been proved for nonlinear wave equation with a mass term. Later the result have been extended to quasi-periodic solutions, but always under the assumption that the mass is strictly positive. So far, in the case of zero mass, existence of periodic solutions has been proved only for a zero-measure set of values of the frequencies and amplitudes of the solutions. I shall discuss a recent result, obtained with V. Mastropietro and M. Procesi, in which the same result can be obtained for a set with large relative Lebesgue measure.

David Gomez-Ullate
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The direct approach to quasi-exact solvability

(Abstract not provided)

Mikael Sonne Hansen
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Topological aspects of a simple model with fractional monodromy

We consider a simple model on $S^2 \times S^2$ with fractional monodromy. We proceed to study some topological/geometrical aspects in the classical, semiclassical and quantum mechanical regimes.

John Harnad

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Dispersionless hierarchies, logarithmic reductions and random matrix asymptotics

We consider rational and logarithmic reductions of the dispersionless 2-Toda hierarchy, and show them to be consistent provided the string equation constraint is imposed. These reductions are also related to random matrix models in the large N limit.

Wu-Yi Hsiang

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Global Geometry of 3-Body Trajectories with Vanishing Angular Momentum

(Abstract not provided)

Jacques Hurtubise

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The geometry of the general Calogero-Moser system

While the Calogero-Moser system for general root systems is staraightforward to define, its global geometry is somewhat elusive. We present an approach that encodes in a natural way the Hamiltonians, and gives a global description of the level sets. (joint work with Eyal Markman and Tom Nevins).

Jens Jonasson

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Quasi-Cauchy-Riemann equations

A system of quasi-Cauchy-Riemann equations (qCR) is a system of first order PDE's of the form $(\det G)^{-1}G\nabla k = (\det \tilde{G})^{-1}\tilde{G}\nabla\tilde{k}$, where $G = \alpha qq^T + \beta q^T + q\beta^T + \gamma$ and $\tilde{G} = \tilde{\alpha}qq^T + \tilde{\beta}q^T + q\tilde{\beta}^T + \tilde{\gamma}$ are elliptic coordinate matrices and k and $\tilde{k}$ are dependent variables. These equations come up naturally as a characterization of a wide class of quasi-potential Newton equations $\ddot{q} = -(\det G)^{-1}G\nabla k = -(\det \tilde{G})^{-1}\tilde{G}\nabla\tilde{k}$, that admit n quadratic integrals of motion and appear to be a natural generalization of the classical separable potential systems when $\tilde{G} = Id$.
For $n = 2$ and a suitable choice of matrices G and $\tilde{G}$, the qCR equations become the well known Cauchy-Riemann (CR) equations. Like the CR equations these equations exhibit a remarkable algebraic structure: its solutions can be multiplied in a similar way as solutions to the CR equations can be multiplied through multiplication of analytic functions.
Other well known equations that are closely related to qCR equations are the Laplace equation, the 2-dimensional wave equation and the Bertrand-Darboux equations characterizing separable potentials. In this lecture I will explain all these connections. I will also discuss the structure of the space of solutions for qCR equations.

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Abdelouahab Kadem

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The relation between the topological structure of affine systems and topological structure of homogeneous systems in low dimension

In this paper, we establish the existence of a relation between the topological structure of the set of controllable affine systems denoted by C_a and topological structure of the set of controllable homogenous systems C_h. We consider the following system

$$\begin{cases} \dot{x} = Ax + u(Dx + b) \\ x \in R^2;\ u \in R;\ b \in R^2 \end{cases} \tag{1}$$

where u is piecewise constant control with values in subset Ω of R, bounded or not; A and D are two matrix real 2×2 . For (1) we associate the following homogeneous system

$$\begin{cases} \dot{x} = (A + uD)x \\ x \in R^2;\ u \in R \end{cases} \tag{2}$$

It is known if (2) is controllable in $R^2 - \{0\}$ and (1) has not a fixed point, then (1) is completely controllable in R^2 [1]. It turns out that in certain cases (1) can be controllable but (2) is not controllable but that these cases are marginal. In R^{10} the set of such pairs is the boundary of the set of controllable pairs. The interior being constituted by pairs for which (2) is controllable. It is also known on the one hand that the set C_h is connexe and its boundary present two types of points boundary [2] on the other hand the set C_a is connexe [3]. The question is: can one establish a relation between the boundary of C_h and the one of C_a?

[1] V. Jurdjevic, G. Sallet, *Controllability properties of affine systems*, Siam. J. Control. Optim, 22 ,3 (1984) pp 501-508.
[2] A. Kadem, *Etude de la structure topologique des systémes homogénes contrôlables*, Maghreb Mathematical Review; Volume 10; Number 2; December 2001; pp 235-254.
[3] A. Kadem, *Etude de la connexité de l'ensemble des systèmes affines contrôlables.* Submitted

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Niky Kamran
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Wave equations in Kerr geometry

We will review some recent developments in the analytic and geometric study of the Dirac and scalar wave equations in the Kerr geometry of a rotating black hole in equilibrium. The topics that will be covered include spectral decompositions of propagators and L^∞_{loc} decay estimates in time. The role of generalized symmetries in the analysis will be emphasized. This is joint work with Felix Finster, Joel Smoller and Shing-Tung Yau.

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Andrew King
School of Mathematics, University of Birmingham,
Birmingham, UK

Rational perturbation methods for Boussinesq approximations in a two layer flow

A perturbation method based upon long wave theory is used to derive Boussinesq equations for the three dimensional evolution of the interface in a two layer flow confined within a horizontal pipeline. The equations derived are non linear and results for travelling waves of periodic, surge and soliton form will be presented.An unusual feature of the system is a singularity which can develop when the lower fluid starts to block the pipe on which thew two layer flow is confined.

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Dmitry Korotkin

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Tau-function on Determinants of Laplace operators in singular metrics over Riemann surfaces

We prove rigorous formulas for regularized determinants of Laplace operator in Stroebel metrics over Riemann surfaces. Our results in particular confirm the Sonoda's formula of 1987, where empirical expression for Laplacian determinants for metrics given by moduli square of holomorphic differential were found. As the main tools we use the Fay's variational formulas for Laplacian determinants in smooth metrics and formulas of analytical surgery for metrics with conical singularities.

Ivan Kossenko
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On Preservation of Action Variables for Satellite Librations in Elliptic Orbit with Account of Solar Light Pressure

Planar librations of a satellite, with its mass center moving in the elliptic orbit, are under consideration. Forces of Solar light pressure assumed to act on the satellite besides gravitational ones.

Rotation of dynamically symmetrical satellite is examined as an unperturbed one. Application of the KAM-theorem directly is impossible because of nonanalyticity of Hamilton's function. Through the reduction of perturbed Hamiltonian system to the sequence of symplectic maps, and with the help of Moser's theorem on invariant curve an existence of invariant tori and preservation of action variables near their initial values are proven. An analysis of the limit case vicinity corresponding to orbit eccentricity being equal to one or close to this value is carried out. Order of perturbation in this case is supposed to be fixed. It occurs that in this case action variables also preserve their values in asymptotically large time intervals.

Vadim B. Kuznetsov
Dept of Applied Maths,
University of Leeds

Well integrable and completely separable

We will introduce two new notions which appear to be useful when classifying integrable systems from the separation point of view. We provide motivations for doing this as well as give examples. The proposed approach to integrability versus separability will lead to a natural class of integral transforms and special functions, which we call H-class. This can be thought of as a natural analogue of Darboux transformations for integral operators.

Edwin Langmann

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Exact results related to the quantum elliptic Calogero-Sutherland system

I discuss exact results related to the quantum elliptic Calogero-Sutherland (eCS) model. I shortly describe how quantum field theory techniques can be used to obtain remarkable identities involving eCS Hamiltonians and elliptic functions, and I will show how such identities can be used to obtain perturbative algorithms to construct the eigenfunctions and eigenvalues of this model.

I present various recent results obtained since January this year.

Sara Lombardo

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Reductions of integrable equations. Dihedral group

We discuss algebraic and analytic structure of rational Lax operators. With algebraic reductions of Lax equations we associate a reduction group - a group of twisted automorphisms of the corresponding infinite dimensional Lie algebra. We present a complete study of dihedral reductions for $sl(2, C)$ Lax operators with simple poles and corresponding integrable equations and give few examples of dihedral reductions for $sl(N, C)$ Lax operators.

Krzysztof Marciniak

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Linköping University, 601 74 Norrköping, Sweden

Geometric reduction of Hamiltonian systems

Given a foliation S of a manifold M, a distribution in M transversal to S and a Hamiltonian system on M we present a geometrical method of reducing this system to a foliation S. We analyse its relation with the classical ideas of Dirac (Dirac reduction) and with the more modern theory of J. Marsden and T. Ratiu. Our method is constructive and "constraint-independent" in the sense that it is invariant with respect to the functions that define the foliation S. One of the consequences of this work is that in case of second class constraints (in the terminology of Dirac)one can perform not only usual Dirac reduction of Poisson (Hamiltonian) system on submanifolds but also other types of reductions.

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Gianni Manno

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On symmetries of the equation of motion of a relativistic particle

The equation of motion of a relativistic particle is considered in the framework of jets of 1-dimensional submanifolds of a Lorentz manifold. It is known that classical symmetries of such an equation preserve geodesics but not necessarily the metric. We present new results on higher symmetries of the equation which have special structures of their generating functions.

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Vladimir S. Matveev

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Quantum integrability of the laplacians
for geodesically equivalent metrics and Lichnerowicz conjecture

I will show that if two metrics given on one manifold share the same geodesics (considered as unparameterized curves), then the Beltrami-Laplace operator of one of these metrics admits commuting differential operators of second order. This allows to construct a big series of new examples of quantum integrable hamiltonian systems. (The above results are joint with Topalov). As a geometric new application I will prove the classical Lichnerowicz (projective) conjecture.

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Rey McLenaghan

Department of Applied Mathematics, University of Waterloo,
Waterloo, ON N2L 3G1, Canada

An invariant classification of the orthogonally separable webs for the Hamilton-Jacobi equation in 3-dimensional Euclidean space

The theory of the algebraic invariants of Killing tensors defined on pseudo-Riemannian spaces of constant curvature under the action of the isometry group is described. The theory is illustrated by the computation of a set of fourteen fundamental invariants of the valence two Killing tensor on 3-dimensional Euclidean space under the group of rigid motions. This set of invariants is employed to obtain a characterization of the eleven orthogonal separable webs for the Hamilton-Jacobi equation and the Schrödinger wave equation in Euclidean space. The results described are joint work with Joshua Horwood and Roman Smirnov.

Hil Meijer

Mathematisch Instituut,
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Numerical normal forms for codim 2 bifurcations of fixed points with at most two critical eigenvalues

We present an algorithm to compute the numerical normal form on the center manifold near a codimension 2 bifurcation of a fixed point of a n-dimensional map. Explicit formulas for fixed points with at most two critical eigenvalues are given. These normal forms can be used to verify nondegeneracy conditions and to predict certain global bifurcations. An example using a 3 dimensional map will be presented.

Robert Milson

Dept. Mathematics and Statistics, Dalhousie University,
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Alignment and classification in Lorentzian geometry

We will describe a dimension-independent theory of alignment in Lorentzian geometry, and its application to the problem of tensor classification. As particular examples, we will consider higher dimensional Weyl tensors and trace-free, third order tensors.

Rick Moeckel

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Finiteness for relative equilibria of the four-body problem

If the masses are positive, there are only finitely many relative equilibria of the Newtonian four-body problem, up to symmetry.

Richard Montgomery
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Fitting Hyperbolic pants to a three-body problem

Consider the three-body problem with an attractive $1/r^2$ potential. We first show that, modulo symmetries, the dynamics for the bounded zero-angular momentum solutions is equivalent to a geodesic flow on the thrice-punctured sphere, or "pair of pants". The sphere is the shape sphere. The punctures are the binary collisions [ChM]. The metric generating the geodesics is the Jacobi-Maupertuis metric. The metric is complete, has infinite area, and its ends, the neighborhoods of the punctures, are asymptotically cylindrical. Our main result is that when the three masses are equal then the metric has negative curvature everywhere except at two points (the Lagrange points). A corollary of this negativity is the uniqueness of the $1/r^2$ figure eight, and an essentially complete symbolic dynamics for describing non-collision solutions.

Souichi Murata
Department of Physics, Nagoya University,
Chikusa-ku, Naogya, 464-8602, Japan

Renormalization group symmetry and gas dynamics

A new scenario in the renormalization group symmetry method is introduced to solve an initial value problem for a system of partial differential equations. As a specific case, We give a exact solution for the adiabatic perfect gas dynamics, which describes a contracting and expanding localized mass of gas.

Jim Murdock
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Ames IA 50011

Some new methods in hypernormal form theory

Improvements will be described for the Churchill-Baider theory of hypernormal forms (or unique normal forms)for vector fields at a rest point using the Campbell-Hausdorff group. The process of hypernormalization can be broken into smaller steps, and the required linear algebra for all of these steps can be done in a systematic way that eliminates many row reductions. An incidental consequence is a concrete approach to the spectral sequence method introduced by Jan Sanders.

Concepciòn Muriel

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C^∞-Symmetries and Euler-Lagrange Equations

We present a generalization of the concept of variational symmetry, based on the new prolongations called $\lambda-$prolongations. We introduce the concept of variational $C^\infty-$symmetry that will lead to new reductions for Euler-Lagrange equations. Some results related to conservation of variational symmetries through successive order reductions are also presented.

Nikolai Nekhoroshev

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Fractional classical monodromy

(Abstract not provided)

Vladimir Novikov

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Perturbative Symmetry Approach
for Differential-Difference equations

(Abstract not provided)

Simone Paleari
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Equipartition times in Fermi-Pasta-Ulam system

(Abstract not provided)

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Av. San Rafael Atlixco 186, Col. Vicentina. Mexico D.F. 09340, Mexico

An anisotropic perturbation of the Kepler problem

We study a 2-body problem given by the sum of the Newtonian potential and an anisotropic perturbation that is a homogeneous function of degree $-\beta$, $\beta \geq 2$. For $\beta > 2$, the sets of initial conditions leading to collisions/ejections and the one leading to escapes/captures have positive measure. We apply the Melnikov method (a variational technique), to show that for every $\beta \neq 2, 3$, the flow on the zero-energy manifold is chaotic. For $\beta = 2$, a case we prove integrable, the infinity manifold of the zero-energy level is a disconnected set, which has heteroclinic connections with the collision manifold.

Len Pismen
Technion - Israel Institute of Technology
Technion City, 32000 Haifa, Israel

Weakly dissipative vortex solitons

Perturbation techniques are applied to dynamics of topological defects in weakly dissipative complex Ginzburg Landau equations. The problems to be considered are particle interactions, possibilities of and obstacles to reduction of the PDE dynamics to particle dynamics, curvature driven motion, and formation of composite structures (bound particles).

Juha Pohjanpelto
Dept of Mathematics, OSU,

Corvallis, OR 97331, USA

Differential Invariants for Pseudogroup Actions on Submanifolds

In this talk I will report on my ongoing joint work with Peter Olver on developing systematic and constructive algorithms for the identification and analysis of various invariants for the action of infinite dimensional pseudogroups.

Let $\mathcal{G}$ be a Lie pseudogroup acting on a manifold M. I will start by employing the variational bicomplex structure of the bundle $\mathcal{G}^\infty \to M$ of infinite jets of transformations in $\mathcal{G}$ to define analogues of the Maurer-Cartan forms for $\mathcal{G}$, from which Cartan's structure equations for $\mathcal{G}$ can be derived. Next let $J^\infty \to M$ stand for the bundle of infinite jets of p-dimensional submanifolds of M and write $\mathcal{H}^\infty$ for the pull-back of the bundle $\mathcal{G}^\infty$ to J^∞. We define a moving frame for $\mathcal{G}$ to be a $\mathcal{G}$ equivariant section of $\mathcal{H}^\infty$. One sees that a pseudogroup $\mathcal{G}$ admits moving frames provided that the induced action of $\mathcal{G}$ on J^∞ is in a suitable sense locally free. I will discuss some important applications of moving frames, in particular, an invariantization process by which one can systematically produce a complete set of differential invariants and an invariant coframe for the action of $\mathcal{G}$ on J^∞, as well as recurrence relations amongst these invariant quantities.

Finally, I will describe a practical algorithm based on Taylor series expansions for constructing various invariants for the action of $\mathcal{G}$.

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Antonio Ponno

Dip. di matematica, Università di Milano
Via Saldini 50 - 20133 Milano - Italy

Energy cascade in FPU models

Some scaling laws characterizing the energy cascade in FPU models are derived making use of canonical perturbation theory (quasi-resonant averaging).

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Giuseppe Pucacco

Dipartimento di Fisica - Università di Tor Vergata
Via della ricerca scientifica, 1 - Roma, Italy

Integrable Hamiltonian systems with polynomial invariants

We investigate integrable 2-dimensional Hamiltonian systems with scalar and vector potentials, admitting strong and weak second invariants which are polynomials in the momenta.

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Orlando Ragnisco
Dipartimento di Fisica, Università di Roma III
Via Vasca Navale 84 - I-00146 Roma, Italy

The "Bruschi-Ragnisco" lattice revised: Classical and Quantum BTs and related interpolating flows

Relying upon the fundamental results found by Y.Suris, the authors consider classical and quantum exact discretizations of the Bruschi-Ragnisco lattice. At the classical level, the interpolating hamiltonian flow is determined; at the quantum level, the corresponding Q-Baxter operator is constructed, and its spectral properties are described.

Giovanni Rastelli
Università di Torino

Remarks on Conformal Killing Tensors and Separation of Variables

Properties of Conformal Killing Tensors are examined in their application to some topics in classical separation of variables theory: L-systems theory and Conformal Separation of the Hamilton-Jacobi equation.

Stefan Rauch-Wojciechowski
Department of Mathematics, Linkoping University
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Solution of the Jacobi problem of separation of variable

Our solution of the Jacobi problem of finding separation variables for natural Hamiltonian systems $H = (1/2)p^2 + V(q)$ is explained. It has a form of an effective criterion that for any given potential $V(q)$ tells whether there exist suitable separation coordinates $x(q)$ and how to find these coordinates, so that the Hamilton-Jacobi equation of the transformed Hamiltonian is separable. The main reason for existence of such criterion is the fact that for separable potentials $V(q)$ all integrals of motion depend quadratically on momenta and that all orthogonal separation coordinates stem from the generalised elliptic coordinates. This criterion is directly applicable to the problem of separating multidimensional stationary Schrödinger equation of quantum mechanics.

Alexey Rosaev
FGUP NPC NEDRA,
Svobody, 8/38 Yaroslavl (Russia)

The stability of the special case of symmetric N-body problem under resonance perturbation

The gravitation interaction between particles is taken into account in the symmetric N-body problem (central configuration model). After that, the resonance perturbation of the particle of the central configuration by distant satellite is considered in the restricted 3-body problem approach. The shift between simple mean motion resonance and parametric resonance zones is derived. This shift depends from mass satellite and central configuration properties. It is shown, that mutual gravitation between particles of the central configuration occur more significant effect on resonance structure, than satellite mass variation. The few possible applications are discussed.

Jan Sanders
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Normal forms: Theory and Practice

(Abstract not provided)

Vasilisa Shramchenko
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"Real doubles" of Hurwitz Frobenius manifolds

New Frobenius structures on the Hurwitz spaces are found. A Hurwitz space is considered as a real manifold; therefore the number of coordinates is twice as large as the number of coordinates used in the construction of Frobenius manifolds on Hurwitz spaces by B. Dubrovin. The branch points of a ramified covering and their complex conjugates play the role of canonical coordinates on the constructed Frobenius manifolds. Corresponding solutions of WDVV equations and $G-$functions are obtained.

Vladimir Skarka

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Variational approach to soliton generation and stability analysis of multidimensional nonlinear Schrödinger equation

Using variational approach, we consider the conditions for the generation of spatiotemporal solitons called "light bullets" characterized by a balance of diffraction and dispersion with respectively spatial and temporal self-focusing saturating nonlinearly. The dynamics of such completely localized self-guided structures is governed by a (3+1)-dimensional nonlinear Schrödinger equation containing one propagation dimension and three transverse dimensions. The variational approach is also applied in order to study generation and dynamics of optical vortex solitons containing topological phase singularity and nonzero angular momentum. A stability analysis provides conditions for formation of symmetric, stable, exceptionally robust self-guided pulses in the final stage of evolution. The simulations show that the pulse above critical energy is trapped provided its initial parameters belong to the range close to the analytically predicted one. The evolution of singular optical pulses towards vortex solitons is also considered.

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Markus Sköldstam
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Analysis of invariant manifolds and of asymptotic motions for the Tippe-Top

The Tippe-top has the form of a truncated sphere equipped with a little peg. When spun fast on its spherical bottom its center of mass rises up after few seconds and the top continues spinning on the peg. This counterintuitive behavior of the Tippe-Top amused many distinguished physicists (Thompson,Pauli, Heisenberg) and has been the subject of several research papers. A commonly accepted model has been presented as late as 1977 in a paper by R.J.Cohen. We study the equations of motion of the Tippe-top in a vector form through a sequence of embedded invariant submanifolds to unveil the structure of the tops phase space. The last manifold consisting of the asymptotic motions, is analyzed completely. A full analysis of stability of asymptotic motions as a function of parameters and of the initial conditions has been performed with the use of the LaSalle invariance principle. It confirms the known physical behavior of the Tippe-Top.

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Roman Smirnov

Department of Mathematics and Statistics, Dalhousie University, Halifax, Canada, B3H 3J5

A new invariant theory: Invariants, covariants and joint invariants of Killing tensors

Multiple factors have contributed to the recent resurgence of interest among mathematicians in the classical invariant theory (CIT) of homogeneous polynomials.

The factor that brought my collaborators and me to the area is the fact that the underlying ideas of CIT could be naturally incorporated into the study of Killing tensors defined in pseudo-Riemannian spaces of constant courvature. The result of this natural fusion is the invariant theory of Killing tensors (ITKT) which shares many essential features with the original CIT.

Of particular interest are the vector spaces of Killing tensors of valence two whose invariants and covariants can be effectively used to solve the problems of the Hamilton-Jacobi theory of orthogonal separation of variables.

I target to review some of the results obtained so far.

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Matteo Sommacal

Sissa

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New many-body problems in the plane with periodic solutions

I will discuss a family of toy models for many-body interactions including velocity-dependent forces. By generalizing a construction due to Calogero, we obtain a class of N-body problems in the plane which have periodic orbits for a large class of initial conditions. The two- and three-body cases (N = 2, 3) are exactly solvable, with all solutions being periodic; for N¿3 Painlevé analysis indicates that the system should not be integrable. The construction can be generalized to a broad class of systems, and the mechanism which describes the transition to orbits with higher periods, and eventually to aperiodic or even chaotic orbits, could be present in more realistic models with a mixed phase space. This scenario is different from the onset of chaos by a sequence of Hopf bifurcations. We will show (as movies) numerical simulations of the trajectories obtanied with a purposely written software. This work was done in cooperation with David Gomez Ullate and Andrew Hone.

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Susanna Terracini

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On some optimal partition problems with applications

In this talk we give an unified approach to some questions arising in different fields of nonlinear analysis, namely: (a) the study of the structure of the Fučík spectrum and (b) possible variants and extensions of the monotonicity formula by Alt–Caffarelli–Friedman. In the first part of the paper we present a class of optimal partition problems involving the first eigenvalue of the Laplace operator. Beside establishing the existence of the optimal partition, we develop a theory for the extremality conditions and the regularity of minimizers. As a first application of this approach, we give a new variational characterization of the first curve of the Fučík spectrum for the Laplacian, promptly adapted to more general operators.

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Marco Antonio Teixeira
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Invariant Varieties of Discontinuous Vector fields

The present deals with reversible vector fields with discontinuous right hand side. The main goal is to study the dynamics around typical singularities of a class of Reversible Discontinuous Vector Fields that are of interest in the Theory of Geometric Control. The basic model on which we start the discussion is: $X_0(x, y, z, w) = (y, z, w, -sgn(x))$ This model, although apparently simple possesses surprisingly a very complicated dynamics near the origin. In addition, this model is "reversible" and has a ("non-smooth") integrable Hamiltonian structure. We consider reversible perturbations of such system. The main result consists in presenting conditions under which there exists invariant varieties filled up by periodic orbits (in the sense of Liapunov Center Theorem)

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Gianpaolo Valente
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Symmetry of singularities and orbit spaces of compact linear groups

In singularity theory, it is well known that the bifurcation diagrams of the simple singularities of type A_k (for $k \geq 1$), D_k (for $k \geq 4$), E_6, E_7 and E_8

are diffeomorphic to the manifolds of the non–regular orbits of the homonymous Coxeter reflection group acting on the complex space. Deep research was devoted to the extension of this connection to other finite reflection groups. We intend to propose an observation which could be useful to identify singularities which could be associated to general compact linear groups (including finite reflection groups). It is based on an equivalence relation introduced in the classification of real orbit spaces of compact (coregular) linear groups, in the search for universality properties in spontaneous symmetry breaking. The connections with the classification of stable singularities of smooth maps and with the classification of singularities with additional structures such as boundaries and symmetries are also examined.

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Gianmaria Verzini

Politecnico di Milano - Dipartimento di Matematica
P.zza Leonardo da Vinci 32, 20133 Milano

Optimal partitions problems and competing species systems

Some recent results, obtained in collaboration with Monica Conti and Susanna Terracini, are presented. The results regard a class of optimal partition problems arising as asymptotic limits of competing species systems when the interaction is arbitrarily strong (segregated states). Applications to the Fucik spectrum for elliptic differential equations, and to some monotonicity formulae, are shown.

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Irina Yehorchenko

Institute of Mathematics of the National Academy of Sciences of Ukraine
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Differential invariants for infinite-dimensional algebras

We present an approach for construction of functional bases of differential invariants for some infinite-dimensional algebras with coefficients of generating operatorsdepending on arbitrary functions. Some examples are presented,including invariants for algebras of equivalence groups for some PDE classes.

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Boris Zhilinskii

MREID, Universite du Littoral,

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Interpretation of quantum Hamiltonian monodromy in terms of lattice defects

The analogy between monodromy in dynamical (Hamiltonian) systems and defect in crystal lattices is used in order to formulate geberal conjectures about possible types of qualitative features of quantum systems which can be interpreted as a manifestation of classical monodromy in quantum finite particle (molecular) problems.