SPT2018
Talks & Abstracts

S. Margherita di Pula, 3–10 June 2018

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Luca Biasco (Roma)

KAM Theory for secondary tori

We deal with secondary tori, namely the tori which appear close to resonances in Hamiltonian systems. In particular we give some results on their density in mechanical systems for generic analytic potentials. [Joint work with L. Chierchia]

Sergey Bolotin (Moscow & Madison)

Topology, singularities and integrability in Hamiltonian systems

We consider the problem of the existence of polynomial in momenta first integrals of natural Hamiltonian systems with two degrees of freedom on a fixed energy level (conditional Birkhoff integrals). Suppose that the potential energy V on the configuration space M has singular points of type $V(q) \simeq |q|^{\alpha_j}$. We prove that in the presence of Birkhoff conditional integrals on an energy level $H = h > \sup_M V$, we have $\sum \alpha_j \leq 2\chi(M)$. Positiveness of the topological entropy is also proved. Generalizations to systems with 3 degrees of freedom are discussed. This is a joint work with V. Kozlov (Moscow Mathematical Institute).

Henk Broer (Groningen)

A Galilean dance: on the dynamics of Jupiter and his Galilean moons

In 1610 Galileo reports on four moons (meanwhile called Io, Europa, Ganymedes and Callisto) around the planet Jupiter. He interpreted this as a miniature solar system and as a support of a heliocentric world view. In 1799 Laplace postulates a $1 : 2 : 4$ orbital resonance in in the motion of the inner three moons. The problem is how to fit this into a Newtonian (or relativistic) description of the system. The first to deal with the mathematics of this was Willem de Sitter, a PhD student of the astronomer J.C. Kapteyn, in the beginning of the 20th century. He found a resonant periodic motion of the inner three moons Io, Europa and Ganymedes . In the talk we also incorporate the outer moon Callisto in the considerations and, apart from periodic motions also find lirbrational multi-periodic motions of 8 frequencies. These theoretical results may well give more possibilites interpret observational data.

Francesco Calogero (Roma)

Zeros of polynomials and solvable nonlinear evolution equations

A short (about 170-page) book of mine with this title is now (25.01.2018) in press with Cambridge University Press, and some related developments are provided in the following papers written subsequently: F. Calogero, "Zeros of entire functions and related systems of infinitely many nonlinearly coupled evolution equations", Theor. Math. Phys. (in press); F. Calogero and F. Leyvraz, "Examples of Hamiltonians isochronous in configuration space only, and their quantization", J. Math. Phys. (submitted); O. Bihun and F. Calogero, "Time-dependent polynomials with one double root, and related new solvable systems of nonlinear evolution equations", Qual. Theory Dyn. Syst. (submitted); F. Briscese and F. Calogero, "Isochronous solutions of Einstein's equations and their Newtonian limit", Int. J. Geom. Meth. Modern Phys. (submitted); F. Calogero, "Finite and infinite systems of nonlinearly coupled ordinary differential equations the solutions of which feature remarkable Diophantine findings", J. Nonlinear Math. Phys. (submitted). I plan to report tersely some of these findings (including some of those in the book), and to discuss them with interested participants during the meeting.

Sandra Carillo (Roma)

Solutions of KdV and mKdV non-commutative equations

(Joint work with Mauro Lo Schiavo and Cornelia Schiebold) Results concerning nonlinear non-Abelian KdV and mKdV equations are presented. Specifically, on the basis of recent results, matrix solutions of the KdV and mKdV equations are constructed: a key role is played by the link, via Bcklund Transformation, which relates these equations. Matrix mKdV solutions seems to be new. The latest achievements are presented and briefly discussed. The Hamiltonian structure, well established in the scalar case, is currently under investigation.

Raffaele Carlone (Napoli)

NLS in d=2 with concentrated nonlinearities

We consider a two-dimensional nonlinear Schrödinger equation with concentrated nonlinearity. In both the focusing and defocusing case we prove local well-posedness, i.e., existence and uniqueness of the solution for short times, as well as energy and mass conservation. In addition, we prove that this implies global existence in the defocusing case, irrespective of the power of the nonlinearity, while in the focusing case we prove the existence of blowing-up solutions.

Luigi Chierchia (Roma)

On Kolmogorov's set in classical mechanical systems

Classical KAM Theory deals with the persistence of Lagrangian tori that are deformations of unperturbed maximal Diophantine tori. We consider the full Kolmogorov's set for classical mechanical/Newtonian Hamiltonian systems, namely, the set of all analytic, Diophantine, Lagrangian invariant tori (including, in particular, the secondary tori arising for effect of the perturbation), and discuss its topology and measure. [Joint work with L. Biasco]

Martina Chirilus-Bruckner (Leiden)

Localized structures in an extended Klausmeier model

The evolution of vegetation patterns in semiarid regions can be described by a systems of reaction-diffusion equations for biomass and water. Despite its simplicity this model features many of the pattern forming phenomena observed in nature and presents a valuable tool to understand the formation and reversing of desertification. We show an extension of the model where the variation of the terrain is taken into account via spatially varying coefficients. Using a novel mix of geometric singular perturbation theory and exponential dichotomies, we show a new construction of localized vegetation patches and discuss their stability and interactions. This is joint work with Arjen Doelman and Robbin Bastiaansen.

Roberto De Leo (Washington)

Topology and Dynamics of quasiperiodic functions

Quasiperiodic functions appear naturally in several pure and applied fields such as the theory of completely integrable Hamiltonian systems, of solitonic solutions of nonlinear PDEs, of quasicrystals and of conductivity in normal metals. Their study is strictly related to the study of foliations induced on manifolds by closed 1-forms and major contributions to this field were given over the last forty years by S.P. Novikov and V.I. Arnold and their respective schools. In this talk we present the main pure and applied results in this field.

Antonio Degasperis (Roma)

Integrability and linear instabilities of two coupled continuous waves

We consider the integrable coupling of two nonlinear Schroedinger equations and the linear stability properties of their continuous wave solutions. Starting from the Lax pair we construct the eigenmodes of the linearized equation as defined on an appropriate spectrum in the complex plane. Assessing the stability against small localized perturbations requires first the construction of this spectrum and then the computation of the (complex) frequency of the eigenmodes. We do this by both analytical and numerical methods for both self- and cross-couplings, and for all values of the wave parameters. The method is general enough to be applicable to a large class of integrable systems.

Gianne Derks (Guildford)

One-dimensional periodic solutions in a three-component reaction-diffusion system

Periodic patterns occur ubiquitously in nature, but the mechanism behind the formation of periodic patterns away from onset is not well understood. In this talk we consider the mechanism behind the generation of periodic stationary solutions in a singularly perturbed reaction-diffusion system. The system has one fast nonlinear component, interacting with two slow components. We investigate the existence and bifurcations of families of one-dimensional periodic solutions in this system. It will be shown how changes in the slow manifold and changes in the fast dynamics lead to an intriguing sequence of self-replicating patterns of large amplitude periodic waves. We will conclude this talk with a discussion about extensions to two-dimensional patterns and travelling waves. (Joint work with Peter van Heijster and David Lloyd)

Maciej Dunajiski (Cambridge)

Solitons on wormholes

The sine–Gordon equation in 1+1 dimensions admits a static kink solution with with topological charge 1. The kinks do not exist (for neither SG nor any other scalar field theory) in flat d+1 dimensions, where d¿1. I shall prove the existence of static kinks on 3+1 dimensional curved wormhole space-time with two assymptoticaly flat regions, and discuss linear and non–linear stability of the kinks. This is joint work with Piotr Bizon, Gary Gibbons and Michal Khal.

Konstantinos Efstathiou (Groningen)

Monodromy and Circle Actions

Standard Hamiltonian monodromy was introduced by Duistermaat as an obstruction to the existence of global action-angle coordinates in integrable Hamiltonian systems [1]. It refers to the monodromy of torus bundles that typically exist in such systems. Fractional Hamiltonian monodromy, introduced by Nekhoroshev, Sadovski, and Zhilinski in [2], generalizes standard monodromy by considering not only torus bundles but also more general fibrations with singular fibers. In this talk I present results concerning both standard and fractional monodromy that were recently obtained in collaboration with Nikolay Martynchuk [3, 4]. It turns out that, in integrable Hamiltonian systems with a Hamiltonian circle action, both standard and fractional monodromy can be solely determined through a careful study of the fixed points of the circle action and their weights. A basic ingredient of this approach is the definition of generalized parallel transport of homology cycles introduced in [5]. These results will be demonstrated in several examples of integrable Hamiltonian systems. See: [1] J. J. Duistermaat, “On global action-angle coordinates”, Comm. Pure Appl. Math. 33 (1980), 687706; [2] N. N. Nekhoroshev, D. A. Sadovski, and B. I. Zhilinski, “Fractional Hamiltonian Monodromy”, Ann. H. Poincar 7 (2006), 10991211; [3] K. Efstathiou and N. Martynchuk, “Monodromy of Hamiltonian systems with complexity 1 torus actions”, J. Geom. Phys. 115 (2017), 104115; [4] N. Martynchuk and K. Efstathiou, “Parallel Transport along Seifert Manifolds and Fractional Monodromy”, Comm. Math. Phys. 356 (2017), 427449; [5] K. Efstathiou and H. W. Broer, “Uncovering fractional monodromy”, Comm. Math. Phys. 324 (2013), 549588.

Filippo Giuliani (Roma)

On the integrability of the Degasperis-Procesi equation: control of Sobolev norms and Birkhoff resonances

In 2002 Degasperis-Holm-Hone proved that the Degasperis-Procesi equation possesses a Lax pair and consequently infinitely many conserved quantities. It is well known that these constants of motion do not control the H^1 norm and the equation accomodates peakon solutions and breaking wave phenomena. In this talk I will show that, when the dispersion terms of the Degasperis-Procesi equation are not naught, these constants of motion are analytic on some neighborhood of the origin of all H^s-Sobolev spaces and they control the Sobolev norms of the solutions with small initial data. I will also discuss a result on the Birkhoff normal form obtained by exploiting the particular algebraic structure of the conserved quantities. This is a joint work with R.Feola and S. Pasquali.

Emanuele Haus (Napoli)

Time quasi-periodic gravity water waves in finite depth

We prove the existence and the linear stability of Cantor families of small amplitude time quasi-periodic standing water wave solutions - i.e. periodic and even in the space variable x - of a bi-dimensional ocean with finite depth under the action of pure gravity. Such a result holds for all the values of the depth parameter in a Borel set of asymptotically full measure. This is a small divisor problem. The main difficulties are the quasi-linear nature of the gravity water waves equations and the fact that the linear frequencies grow just in a sublinear way at infinity. To overcome these problems, we first reduce the linearized operators obtained at each approximate quasi-periodic solution along the Nash-Moser iteration to constant coefficients up to smoothing operators, using pseudo-differential changes of variables that are quasi-periodic in time. Then we apply a KAM reducibility scheme that requires very weak Melnikov non-resonance conditions (which lose derivatives both in time and space), which we are able to verify for most values of the depth parameter using degenerate KAM theory arguments. This is a joint work with P. Baldi, M. Berti and R. Montalto.

Renato Huzak (Hasselt)

Slow-fast bifurcations and Hilbert's 16th problem

The goal of our talk is to study limit cycles in slow-fast and regular codimension 4 saddle-node bifurcations. We use geometric singular perturbation theory and the family blow-up. Using the blow-up techniques we reduce the codimension of the system and then we use the well known results about slow-fast Bogdanov-Takens bifurcations, slow-fast Hopf bifurcations and slow-fast codimension 3 saddle and elliptic bifurcations.

Nabil Kahouadji (Chicago)

Isometric Immersions of Pseudo-Spherical Surfaces via PDEs

Pseudo-spherical surfaces are surfaces of constant negative Gaussian curvature. A way of realizing such a surface in 3d space as a surface of revolution is obtained by rotating the graph of a curve called tractrix around the z-axis (infinite funnel). There is a remarkable connection between the solutions of the sine-Gordon equation $u_{xt} = \sin u$ and pseudo-spherical surfaces, in the sense that every generic solution of this equation can be shown to give rise to a pseudo-spherical surface. Furthermore, the sine-Gordon equation has the property that the way in which the pseudo-spherical surfaces corresponding to its solutions are realized geometrically in 3d space is given in closed form through some remarkable explicit formulas. The sine-Gordon equation is but one member of a very large class of differential equations whose solutions likewise define pseudo-spherical surfaces. These were defined and classified by Chern, Tenenblat and others, and include almost all the known examples of "integrable" partial differential equations. This raises the question of whether the other equations enjoy the same remarkable property as the sine-Gordon equation when it comes to the realization of the corresponding surfaces in 3d space. We will see that the answer is no, and will provide a full classification of hyperbolic and evolution equations. The classification results will show, among other things, that the sine-Gordon equation is quite unique in this regard amongst all integrable equations. [1] N. Kahouadji, N. Kamran and K. Tenenblat, Local isometric immersions of pseudospherical surfaces and k-th order evolution equations, arXiv:1701.08004 [math.DG] (Jan 27, 2017) [2] N. Kahouadji, N. Kamran and K. Tenenblat, Second-order Equations and Local Isometric Immersions of Pseudo-spherical Surfaces, Comm. Anal. Geom., 24, (2016), no. 3, 605- 643. [3] N. Kahouadji, N. Kamran and K. Tenenblat, Local Isometric Immersions of Pseudospherical Surfaces and Evolution Equations, Fields Inst. Commun., 75, (2015), 369-381.

Boris Konopeltchenko (Lecce)

Regularization of the gradient catastrophes for the Burgers-Hopf hierarchy and Jordan chain

Non-standard parabolic regularization of the gradient catastrophes of all orders for the Burgers-Hopf equation is proposed. It is based on the step by step regularization of them by embedding the Burgers-Hopf equation into integrable multi-component parabolic systems of quasi-linear PDEs with the most degenerate Jordan blocks. Probabilistic realization of such procedure is presented. The complete regularization is achieved by the embedding into the infinite parabolic Jordan chain. It is shown that the Burgers equation is a particular reduction of the Jordan chain.

Niclas Kruff (Aachen)

Coordinate-independent criteria for Hopf bifurcations

We discuss the occurrence of Poincar-Andronov-Hopf bifurcations in parameter dependent ordinary differential equations, with no a priori assumptions on special coordinates. The first problem is to determine critical parameter values from which such bifurcations may emanate, a solution for this problem was given by W.-M. Liu. We add a few observations from a different perspective. Then we turn to the second problem, viz., to compute the relevant coefficients which determine the nature of the Hopf bifurcation. As shown by J. Scheurle and co-authors, this can be reduced to the computation of Poincar-Dulac normal forms (in arbitrary coordinates) and subsequent reduction, but feasibility problems quickly arise. We present a streamlined and less computationally involved approach to the computations. The efficiency and usefulness of the method is illustrated by examples. See: https://arxiv.org/abs/1708.06545

Jaume Llibre (Barcelona)

On the real Jacobian conjecture

The real Jacobian conjecture in R^2 states: If $F = (f, g) : R^2 \to R^2$ is a polynomial map such that $\mathtt{det}(DF(x))$ is different from zero for all $x \in R^2$, then F is injective. The real Jacobian conjecture had a negative answer by Pinchuk in 1994. Now several authors look for adding an additional assumption to the fact that $\mathtt{det}(DF(x))$ is different from zero for all $x \in R^2$, in order that the conjecture holds. We shall also talk about the Jacobian conjecture in R^2 which states: If $F = (f, g) : R^2 \to R^2$ is a polynomial map such that $\mathtt{det}(DF(x))$ is constant and different from zero for all $x \in R^2$, then F is injective. The Jacobian conjecture remains open. We will present the following two results which under some additional assumption both conjectures hold. *Theorem 1.* Let $F = (f, g) : R^2 \to R^2$ be a polynomial map such that $\mathtt{det}(DF(x))$ is different from zero for all $x \in R^2$. We assume that the degrees of f and g are equal and that the higher homogeneous terms of the polynomials f and g do not have real linear factors in common, then F is injective. *Theorem 2.* Let $F = (f, g) : R^2 \to R^2$ be a polynomial map such that $\mathtt{det}(DF(x))$ is different from zero for all $x \in R^2$ and $F(0,0) = (0,0)$. If the higher homogeneous terms of the polynomials $ff_x + gg_x$ and $ff_y + gg_y$ do not have real linear factors in common, then F is injective. Theorems 1 and 2 are proved using qualitative theory of the ordinary differential equations in the plane. We shall present the key points of their proofs.

Antonella Marchesiello (Praha)

Superintegrable 3D systems in a magnetic field and separation of variables

We study the problem of the classification of three dimensional superintegrable systems in a magnetic field in the case they admit integrals polynomial in the momenta, two of them in involution and at most of second order (besides the Hamiltonian). Both the classical and quantum case are considered, as it is known that already in two dimensions, when a magnetic field is present, classical and quantum integrable systems do not necessarily coincide. We start by considering second order integrable systems that would separate in subgroup-type coordinates in the limit when the magnetic field vanishes. We look for additional integrals which make these systems minimally or maximally superintegrable. We show that the leading structure terms of the second order integrals responsible for integrability should be considered in a more general form than for the case without magnetic field. Joint work with L. nobl and P. Winternitz.

Krzysztof Marciniak (Linkoeping)

Time-dependent deformations of Frobenius integrable Hamiltonian systems

Motivated by the theory of Painlevé equations and associated hierarchies, we study non-autonomous Hamiltonian systems that are Frobenius integrable. In particular, we establish sufficient conditions under which the Hamiltonians for these systems can be obtained as polynomial-in-times deformations of explicitly time-independent Hamiltonians constituting a finite-dimensional Lie algebra with respect to the Poisson bracket.

Jessica Massetti (Roma)

On the existence of almost periodic solutions for the Nonlinear Schroedinger equation

The problem of persistence of invariant tori in infinite dimension is a challenging problem in the study of PDEs. There is a rather well established literature on the persistence of n-dimensional invariant tori carrying a quasi-periodic Diophantine flow (for one-dimensional system) but very few on the persistence of infinite-dimensional ones. Inspired by the classical "twisted conjugacy theorem" of M. Herman for perturbations of degenerate Hamiltonians possessing a Diophantine invariant torus, we intend to present a compact and unified frame in which recover the results of Bourgain and Poeschel on the existence of almost-periodic solutions for the Nonlinear Schroedinger equation. We shall discuss the main advantages of our approach as well as new perspectives. This is a joint work with L. Biasco and M. Procesi.

M. Concepcion Muriel Patino (Cadiz)

Combined use of solutions of the determining equations for commuting symmetries and $\lambda-$symmetries

Some relationships between (generalized) commuting symmetries and $\lambda-$symmetries for second-order ordinary differential equations are established. The combined use of solutions of the respective determining equations permits to obtain solutions of several equations related to the differential operator associated to the equation. Such solutions are used to obtain important objects for the integrability of the equations: first integrals, integrating factors and Jacobi last multipliers. Several examples illustrate the presented procedures.

Peter J. Olver (Minneapolis)

Reassembly of broken objects

The problem of reassembling broken objects appears in a broad range of applications, including jigsaw puzzle assembly, archaeology (broken pots and statues), surgery (broken bones and reassembly of histological sections), paleontology (broken fossils and egg shells), and anthropology (more broken bones). I will discuss recent progress on such problems, based on advances in the mathematical apparatus of transformation groups and groupoids, symmetry and equivalence problems, moving frames, invariant signatures, and invariant numerical approximations.

Gianluca Panati (Roma)

Spin conductance and conductivity in insulators: an overview

Our long-term research project aims at investigating spin transport in 2-dimensional insulators, with the goal of establishing whether any of the transport coefficients corresponds to the Fu-Kane-Mele topological index which characterizes 2d time-reversal-symmetric topological insulators. The first part of the project consists in a new, first-principle derivation – based on techniques originating from space-adiabatic perturbation theory – of a Kubo-like formula for charge and spin conductivity in gapped periodic quantum systems. As far as charge transport is concerned, the result consists in a new derivation, which does not rely on the usual Linear Response Ansatz, of the usual Kubo formula for charge conductivity. When spin transport is considered, our new Kubo-like formula exhibits additional terms with respect to a naif generalization of the usual Kubo formula from charge to spin transport. The physical relevance of these new terms is now under investigation. (This first part of the project is a joint work with G. Marcelli, D. Monaco, and S. Teufel). The second part of the project investigates instead the relation between the spin conductivity σ^{s_z} and the spin conductance G^{s_z}, a relation which is not trivial in view of the subtleties of spin transport. In a recent preprint (joint work with G. Marcelli and C. Tauber) we focus on the Kubo-type terms (thus neglecting the additional terms mentioned above) and we prove that for any gapped, periodic, near-sighted discrete Hamiltonian, the Kubo spin conductivity and spin conductance are mathematically well-defined and the equality $\sigma_K^{s_z} = G_K^{s_z}$ holds true. Moreover, we argue that the physically relevant condition to obtain the equality above is the vanishing of the mesoscopic average of the spin-torque response, which holds true under our hypotheses on the Hamiltonian operator. This vanishing condition might be relevant in view of further extensions of the result, e. g. to ergodic random discrete Hamiltonians or to Schrödinger operators on the continuum.

Chara Pantazi (Barcelona)

Quadratic systems having a singular cubic curve invariant.

We consider planar polynomial systems of degree two having a singular cubic curve invariant. First we present a complete classification of such systems. Then we present the bifurcation diagrams and the global phase portraits in the Poincare disk. We also present the topological classification of such systems. An additionally information about its integrability is also provided. This work is in collaboration with Dr. J. Llibre.

Stefano Pasquali (Roma)

Dynamics of the nonlinear Klein-Gordon equation in the nonrelativistic limit

In this talk we present some results about the nonrelativistic limit ($c \to \infty$) of the nonlinear Klein-Gordon (NLKG) equation on M, where M is either R^d or a d-dimensional smooth compact manifold. The problem has been extensively studied over more than thirty years, and essentially all known results show convergence of solutions of the NLKG equation to solutions of the nonlinear Schrödinger (NLS) equation for times of order $O(1)$, locally uniformly in time. By combining techniques of Hamiltonian perturbation theory and results within the framework of dispersive PDEs, we perform r steps ($r \geq 1$) of normal form procedure in order to construct a higher-order normalized approximation of NLKG (which corresponds to the NLS at order $r = 1$), and we prove that solutions of the approximating equation approximate solutions of the NLKG equation locally uniformly in time. Furthermore, we prove that when $r \geq 2$ and $M = R^d$, $d \geq 2$, small radiation solutions of the order $-r$ normalized equation approximate solutions of the NLKG up to times of order $O(c^{2(r-1)})$.

Gabriella Pinzari (Padova)

The two-centre problem vs the three–body problem

The two-centre problem is the problem of the motion of a moving particle in the gravitational generated by two fixed masses. Euler demonstrated that the Hamiltonian of the two-centre problem has two effective degrees of freedom both in the spatial and the planar case, and it can be integrated by separation of variables. However, the solution is highly involved, and, recently, many authors have studied the problem from different point of views, in order to extract qualitative properties. We discuss an approach based on perturbation theory and possible applications to the three–body problem.

Michela Procesi (Roma)

Nekhoroshev estimates for the NLS equation on the circle

I will discuss a recent result with L. Biasco and J. Massetti on stability times for NLS equations on the circle. We discuss two main cases: the stability of the Sobolev norm over exponentially long times and the stability of Gevray norm over subexponential times. Our result is obtained by proving a rather abstract Birkhoff normal form theorem for analytic Hamiltonians of scales of Hilbert spaces.

Giuseppe Pucacco (Roma)

The dynamics of the "de Sitter resonance"

We describe a Hamiltonian model for the equilibria of a multi-resonant 1+3 body gravitating system and compare it with the dynamics of the inner Galilean satellites and of resonant exo-planetary systems.

Orlando Ragnisco (Roma)

Symmetry Algebra for Classical and Quantum Perlick Systems

The symmetry algebra fulfilled by the integrals of motion of the maximally superintegrable systems belonging to the Perlick's family will be derived and discussed both in the classical and in the quantum setting.

Stefan Rauch (Linkoeping)

Understanding reversals of a Rattleback

The rattleback is a rigid body having a boat like shape (modelled as a bottom half of an 3-axial ellipsoid) having asymetric (chiral) distribution of mass. When the rattleback is spinned on its bottom in the "wrong" direction then it starts to rattle, it slows down and acquires rotation in the opposite, preferred sense of spinning. This behavior defies our intuition about conservation of angular momentum as the force and the torque responsible for changing the angular momentum and the direction of spinning is not obvious. Majority of papers on the rattlebacks motion study the dependence of stability for spinning solutions, on the sense of rotation, on the shape of rattlebacks surface and on the distribution of mass. There has been no simple intuitive explanation of the counterintuitive rattleback behavior in terms of physical forces and torques. This question is a subject of our paper with M.Przybylska published recently in a journal of Steklov Mathematical Institute of Russian Academy of Sciences [1]. In this paper we study motion of a toy rattleback by using frictionless Newton equations of motion for a rigid body rolling without sliding in a plane. In these equations it is the reaction force of the supporting surface that is the source of the torque turning the rattleback in the preferred sense of rotation. The picture is, however, more subtle as it appears that the direction of the torque depends on the initial conditions and a frictionless, low energy rattleback admits reversals in both directions. I will present simple, intuitive understanding of how the rattlebacks motion depends on initial conditions and will discuss how it is confirmed by simulations of rattlebacks equations for tapping and spinning initial conditions. Simulations show also that long time behavior of such rattleback is, for low energy initial conditions, quasi-periodic and there are inifinitely many reversals in both directions in agreement with truncated equations of Markeev [2]. [1] Stefan Rauch-Wojciechowski, Maria Przybylska "Understanding reversals of a rattleback", Regular and Chaotic Dynamics, July 2017, Volume 22, Issue 4, pp 368-385; [2] A.P.Markeev, "On the dynamics of a solid on an absolutely rough plane", J.Appl. Math. and Mech. 1983, Vol.47, 473-478

David Rojas (Granada)

Resonance of isochronous oscillators

Consider an oscillator with equation $\ddot{x}+V'(x)=0, x \in R$ and assume that it has an isochronous center at the origin. This means that $x=0$ is the only equilibrium of the equation and the remaining solutions are periodic with a fixed period say $T=2\pi$. We are interested in the phenomenon of resonance for periodic perturbations. More precisely, we ask for the class of 2π-periodic function $p(t)$ such that all the solutions of the non-autonomous equation $\ddot{x}+V'(x)=\epsilon p(t)$ are unbounded. Here $\epsilon \neq 0$ is a small parameter. The simplest isochronous center is produced by the harmonic oscillator, $V(x)=(1/2)n^2x^2$, $n=1,2,\dots$ In this case the previous question has a well-known answer: resonance occurs whenever the integral $I_n(p) := \int_0^{2\pi} p(t)e^{int}dt$ does not vanish. After this example the study of resonance for general isochronous oscillators seems natural. As far as we know this question was first raised by Prof. Roussarie in a meeting held in Lleida in II Symposium on Planar Vector Fields. Concrete examples of functions $p(t)$ producing resonance where presented in [Ortega]. See also [Bonheure]. The goal of this work is to identify a general class of forcings leading to resonance. Our main result can be interpreted as a nonlinear version of the condition $I_n(p) \neq 0$. The result we present is a sufficient conditions for resonance but it is not too far from being also necessary: a partial converse of the resonance result holds. See R. Ortega, Periodic perturbations of an isochronous center. Qualitative Theory of Dynamical Systems 3 (2002) 83-91. D. Bonheure, C. Fabry, D. Smets, Periodic solutions of forced isochronous oscillators at resonance. Discrete Contin. Dyn. Syst. 8 (2002) 907-930.

Olga Rossi (Ostrava)

The variational multiplier problem for PDEs

The variational multiplier problem was formulated and solved for the case of a system of two ordinary second order differential equations by Jesse Douglas in his celebrated paper in TAMS in 1941. Since that time, many efforts have been made to elaborate generalizations to systems of three and more SODEs, and a few attempts have been made to consider also PDEs. Contrary to the original Douglas' approach who used purely methods of classical analysis, it turned out that jet geometry appears to be the most efficient tool to tackle this hard problem. In this talk I will refer some recent results which include a solution of this problem for ODEs and new ideas concerning generalization to PDEs.

Adrian Ruiz Servan (Cadiz)

Use of a solvable pair of variational $\mathcal{C}^\infty$-symmetries to reduce the order of Euler-Lagrange equations

A method to reduce by four the order of Euler-Lagrange equations associated to n^{th}-order variational problems is presented. The method consists on using a pair of variational $\mathcal{C}^\infty$-symmetries whose commutator satisfies certain solvability condition. After the performed reduction of order, a (2n-2)-paramater family of solutions for the original Euler-Lagrange equation can be reconstructed by solving two first-order ordinary differential equations.

Andrea Sacchetti (Modena)

Bifurcation trees in nonlinear Schroedinger equations

In this talk we discuss some recent results for a class of nonlinear models in Quantum Mechanics. In particular we focus our attention to the nonlinear one-dimensional Schroedinger equation with a periodic potential and a Stark- type perturbation. In the limit of large periodic potential the Stark-Wannier ladders of the linear equation become a dense energy spectrum because a cascade of bifurcations of stationary solutions occurs when the ratio between the effective nonlinearity strength and the tilt of the external field increases.

Alexey Samohin (Moscow)

Reflection and refraction of a soliton in layered media

The solitons in a layered medium are described by the KdV-Burgers equation $u_t(x,t) = \gamma(x)u_{xx}(x,t) - 2u(x,t)u_x(x,t) + u_{xxx}(x,t)$ related to the viscous and dispersive medium. Note that $\gamma(x) = 0$ corresponds to the KdV equations whose travelling waves solutions are solitons and $\gamma(x) > 0$ corresponds to the KdV-Burgers equation. A soliton solution of the KdV equation, meeting a layer with dissipation, transforms somewhat similarly to a ray of light in the air crossing a semi-transparent plate. We consider two possibilities for $\gamma(x)$: a Π-form density of viscosity; and a smooth function with numerically compact support. The dissipation predictably results in reducing the soliton amplitude, but other effects occur: after the wave leaves the dissipative barrier it retains, on the whole, a soliton form yet a reflection wave arises as small and quasi-harmonic oscillations (a breather). This breather spreads as the soliton is moving through the barrier, and the breather moves in the opposite direction. The process is modelled in some detail.

Paolo Santini (Roma)

Towards a theory of rogue waves in Nature, and the nonlinear Schroedinger model

The focusing Nonlinear Schroedinger (NLS) equation is the simplest universal model describing the modulation instability of quasi monochromatic waves in weakly nonlinear media, the main physical mechanism for the generation of rogue (anomalous) waves (RWs) in Nature. We have recently solved, up to $O(\varepsilon^2)$ corrections, the x-periodic NLS Cauchy problem for an $O(\varepsilon)$ generic initial perturbation of the unstable constant background solution, in the case of a finite number of unstable modes, using the finite gap method, allowing one to derive, in particular, a satisfactory analytic description of the rogue wave dynamics in terms of elementary functions. Matched asymptotic expansions can also be used for this purpose, at least for a small number of unstable modes and under particular conditions. In the simplest case of one unstable mode only, one obtains an exact RW recurrence described by elementary formulas in terms of the initial data. This exact recurrence is expected to become a Fermi-Pasta-Ulam type recurrence in all the physical contexts in which the focusing NLS model applies, and, in a recent experiment on a photorefractive crystal, qualitative and quantitative agreements with the theory have been obtained. Joint work with P. G. Grinevich.

Juergen Scheurle (Munich)

Variational integrators for mechanical systems on a Lie group

In this talk we review some recent developments concerning the discretization of mechanical systems evolving on a Lie group. We focus on the variational approach to derive discrete structure preserving time integrators for the continuous equations of motion. Besides essentials of the general theory specific examples are discussed.

Nitin Serwa (Kent)

Master symmetry for new systems

The study of coherent structures and patterns arising in nonlinear systems is important for the understanding of many phenomena in nature. Therefore a fundamental question arises to determine whether a given system is integrable. One of the criteria used to tackle this problem is through the existence of infinitely many commuting symmetries. Master symmetry is a tool which guarantees the existence of infinitely many symmetries and thus help in determining proof of integrability; moreover, it also can be used for classification of systems. Using the O scheme developed by Prof. J. P. Wang (2015), I would present master symmetries for three new two-component third order Burger type systems (Talati and Turhan, 2016).

Pieralberto Sicbaldi (Granada)

Overdetermined elliptic problems: symmetry and perturbation results

A long-standing problem in Nonlinear Analysis is the classification of the solutions $u : \Omega \to R$ to elliptic overdetermined problems. In the talk I would like to present a brief introduction on this topic and its connections with physical problems. Moreover, I would like to discuss two new results, joint work with D. Ruiz and A. Ros. The first one is a symmetry result, the second one is a perturbation result: (a) If $\Omega \subset R^2$ is diffeomorphic to the half-plane, the existence of a bounded positive solution implies that Ω is a half-plane, for any locally Lipschitz function f. Moreover the solution u must be one-dimensional, i.e. symmetric with respect to any line perpendicular to $\partial\Omega$. (b) For the Schroedinger nonlinearity $f(u) = u^p - u$, there exist nontrivial solutions, defined in domains $\Omega \subset R^n$ that are perturbations of the complement of a round ball. Such solutions, obtained by bifurcation theory, are nonradial, but satisfy some group symmetry, and should appear in Physics in problems modeled by the Schrödinger nonlinear equation. See A. Ros, D. Ruiz and P. Sicbaldi. A rigidity result for overdetermined elliptic problems in the plane. Communications on Pure and Applied Mathematics, 70 (2017), 1223-1252; A. Ros, D. Ruiz and P. Sicbaldi. Solutions to overdetermined elliptic problems in nontrivial exterior domains. Journal of the European Mathematical Society, to appear.

Libor Snobl (Praha)

Spherical type integrable classical systems in a magnetic field

We show that 4 classes of second order spherical type integrable classical systems in a magnetic field exist in the Euclidean space $\mathbf{E}_3$ and construct the Hamiltonian and two second order integrals of motion in involution for each of them. For one of the classes the Hamiltonian depends on 4 arbitrary functions of one variable. This class contains the magnetic monopole as a special case. Two further classes have Hamiltonians depending on one arbitrary function of one variable and 4 or 6 constants, respectively. The magnetic field in these cases is radial. The remaining system corresponds to a constant magnetic field and the Hamiltonian depends on two constants. Questions of superintegrability, i.e. the existence of further integrals, are discussed. Joint work with A. Marchesiello and P. Winternitz.

Joan Torregrosa (Barcelona)

Hilbert numbers using reversible centers

We will show the best lower bounds, that are known up to now, for the Hilbert numbers of polynomial vector fields of degree N, $H(N)$, for small values of N. These limit cycles appear bifurcating from new symmetric Darboux reversible centers with very high simultaneous cyclicity. The considered systems have, at least, three centers, one on the reversibility straight line and two symmetric about it. More concretely, the limit cycles are in a three nests configuration and the total number of limit cycles is at least $2n+m$, for some values of n and m. The new lower bounds are obtained using simultaneous degenerate Hopf bifurcations. In particular, $H(4) \geq 28$, $H(5) \geq 37$, $H(6) \geq 53$, $H(7) \geq 74$, $H(8) \geq 96$, $H(9) \geq 120$, and $H(10) \geq 142$. The talk is based in a joint work with Rafel Prohens: R. Prohens, J. Torregrosa, "New lower bounds for the Hilbert numbers using reversible centers", Preprint 2017.

Cesare Tronci (Guildford)

From Koopman-von Neumann theory to the dynamics of hybrid classical-quantum systems

The border between classical and quantum mechanics remains one of the long lasting unsolved problems of modern physics. Since the early days, the measurement process could only be described by invoking concepts still lacking a mathematical formulation, such as the "quantum decoherence" and the "quantum disturbance" experienced by the classical apparatus. In chemical physics, the necessity of treating some particles as classical and some others as quantum still asks for a consistent theory of mixed classical-quantum dynamics. After revisiting the Koopman-von Neumann theory of classical mechanics from a Hamiltonian viewpoint, this talk exploits a new expression of the Liouville density in terms of classical wave functions in order to formulate a dynamical theory for the interaction between classical and quantum systems. Other than the lack of quantum pure states, classical point particle states are also lost in this picture. In addition, while the quantum density matrix remains positive-definite by construction, the same does not hold for the classical probability density in the presence of classical-quantum correlations.

Ferdinand Verhulst (Utrecht)

Primary and secondary resonance zones in Hamiltonian systems

In Hamitonian systems with 3 or more degrees of freedom it is natural to have both low and high order resonances. Transition of resonance zones provides a tool to discover some of their structure involving different types of resonance. Using the Poincare recurrence theorem and normalization we will discuss a chain of oscillators to demonstrate the ideas and technique. An application will be given to the cubic Klein-Gordon equation on a square with homogeneous Dirichlet conditions.

Jordi Villadelprat (Tarragona)

Bifurcation of zeros in translated families of functions and applications

This talk deals with the study of bifurcation of critical periodic orbits and limit cycles of a family of planar vector fields in a neighbourhood of a polycycle. A key problem in these studies is the breaking of separatrices of the polycycle. The case of cyclicity of hyperbolic polycycles has been extensively studied by Roussarie, Mourtada, Il'yashenko and many others. On the other hand, in our study of the period function of the Loud's centers, we gave a conjecture for the bifurcation of critical periodic orbits. We could not prove it in full generality due in part to some phenomena of breaking of separatrices of the polycycle bounding the period annulus. Our aim is to tackle the simplest setting of breaking one separatrix (or two separatrices in presence of symmetry). Both problems lead to the following type of equation $F(s,\varepsilon;\mu) := F_1(s;\mu) - F_2(s+\varepsilon;\mu) = 0$, where $s = 0$ corresponds to the polycycle, $s \geq 0$ parametrizes the monodromic region, $\varepsilon \approx 0$ is the parameter controlling the breaking of the separatrix and μ are the non-essential parameters of the family. The results that I am going to explain are a joint work, still in progress, with D. Marin (Universitat Autonoma de Barcelona) and P. Mardesic (Universit de Bourgogne).

Irina Yehorchenko (Kiev)

Symmetry, equivalence and reductions of wave equations

We consider a problem of classification of reductions of PDE, noting that the standard Lie procedure does not give a really good classification of reductions. Practical classification requires solving of complicated nonlinear systems, though it appeared possible with a combination of transformation techniques. We present some new general solutions for the multidimensional reduction conditions that are interesting by themselves, such as systems of eikonal, wave and Hamilton-Jacobi equations for different metrics. Relations between symmetries of the initial equation and symmetries of reduction conditions are studied.

Jean Claude Zambrini (Lisboa)

A heat equation based approach to symmetries of SDEs

We shall describe why the study of symmetries of SDEs is made easier if it is based on the ones of parabolic PDEs. And, in particular, the heat equation which already encodes all properties of the Brownian motion.

Marta Zoppello (Padova)

Motion Planning via Reconstruction Theory

In Geometric Control Theory, problems of motion planning consist in finding a control that steers a control system from a starting configuration to a prescribed final one. This problems are common in several fields of research: biology, chemistry, medicine, robotics. In this work we present an approach to study these problems based on the techniques of reconstruction of dynamical systems with symmetry. In particular we focus on a class of control problems, that we called robotic locomotion systems, which are driftless affine control systems with configuration space the total space of a trivial principal fiber bundle $\pi : G \times S \to S$, with G a Lie group and S the shape space which is a manifold. The problem is, for every assigned loop in the base space, to determine the motion of the system on the fiber over the considered loop. This problem is linked to the one of reconstruction in dynamical systems. The main difference is that now the curve on the base manifold is not the integral curve of a (reduced) differential equation, but it is assigned by the controller. The theory of reconstruction under the action of a connected Lie group is well studied from different points of view. Our purpose is to the importance in motion planning of reconstruction results. In particular there are two cases: if the group is compact, the motion on the fiber over a loop is quasi-periodic; if the group is not compact, the motion on the fiber over a loop is either quasi-periodic or there is a drift. The generic case is determined by the group itself. As a consequence some relevant information on the possible motion can be a priori inferred from the structure of the group alone. We present some examples involving some classical Lie groups: $SE(2)$, $SO(2) \times R^2$, $SE(3)$, $SO(3) \times R^3$.